Mechanism Design and Sale of Wireless Spectrum.

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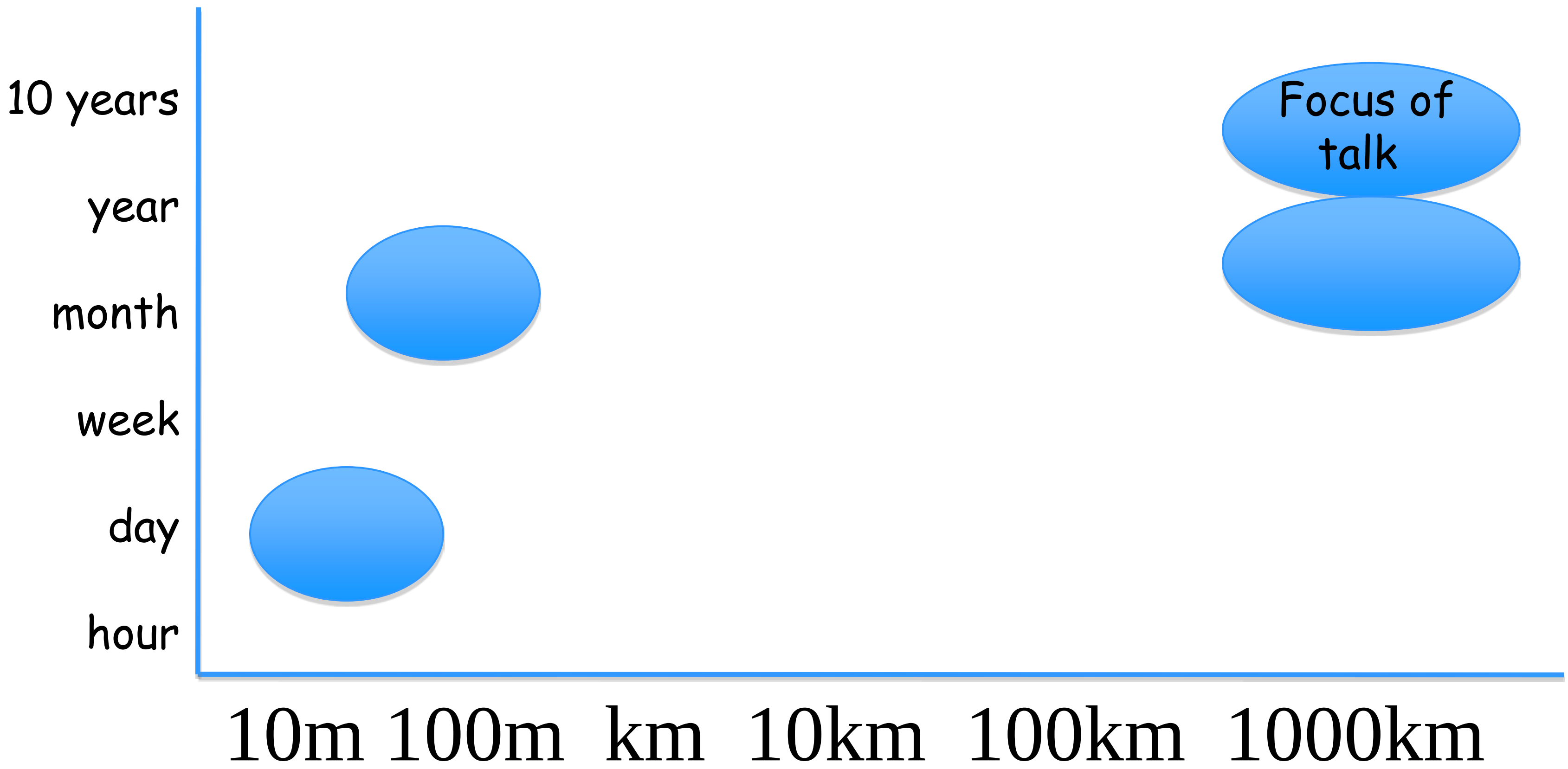
On what spatial and temporal scales might spectrum be traded?



Outline

1. A bit on wireless spectrum auctions in practice (US Auction 73, upcoming white space auction, UK auctions)
2. Intro to theory – auction of a single object (efficiency, incentive compatibility, revenue optimality)
3. Substitute valuations – a utopia for auction theory but can't handle complementarities
4. Complementarities – optimal auctions for single-minded buyers, try extension to heirarchical packages
5. Package clock auction (used in two UK 3G auctions, championed by P. Cramton for next US auction)
6. Profit sharing contracts (as in India's 3G auction) for sale of a single item and correlated private valuations
7. Some directions for future work (+notes and references)

1. FCC Auction 73, 2008

Auction of 700 MHz band licenses in USA
1,099 licenses offered in Auction 73.
January-March 2008   Releasing some of old TV spectrum

All blocks in 700-800 MHz band:

Block A:12MHz	176 licenses	Reserve price $1.8B
Block B:12MHz	734 licenses	Reserve price $1.3B
Block D:10MHz	1 license*	Reserve price $1.3B
Block E:6 MHz	176 licenses (unpaired spectrum) Reserve price $0.9B	

(*Block D is subject to public/private partnership agreement)

Each license: -number of bidding units = upfront payment,
-min opening bid

044-A Knoxville TN, pop. 983,000, 492K bidding units
min opening bid: $720K

068-A Champaign-Urbana, pop. 631,000, 292K bidding units,
min opening bid: $227K

Bidding proceeds in rounds. Bidders place bids on licenses. A *provisional winner* is selected for each license from among the highest bidders.

-- Ties are broken randomly.

-- Carry over previous winner if no new bids.

Anonymity
The identities/owners of the bidders are released before the bidding begins. *No other info is released.*

Activity rule ("use it or lose it"). Bidders must use at least 80% of their eligibility (based on bidding units) in each round, or reduce eligibility.

Percent increases to 95% later in auction "Stage 2"

Exposure problem:

Bidder may want to acquire a bundle of licenses, but may have little interest in a partial bundle.
i.e. *complementary* licenses

Bidder may find it impossible to win, but get stuck with a partial package.

Early rounds offer price discovery--reduces chance of exposure.

FCC Auction 73 package bid option reduced exposure problem:

Block C : 22MHz of spectrum, broken into 12 regions:

1-6 six regions in lower 48 states
7 Alaska
8 Hawaii

Also grouped into three packages

10 Puerto Rico, US Virgin Islands
12 Gulf of Mexico

9 Guam, Northern Mariana Islands
11 American Somoa

Q. When could the package option make a difference?

A. One strong bidder desiring nationwide C band license, but much less interested in part of nation, versus a second, very determined bidder, who wants to cover half of nation. First bidder can bid on package aggressively without fearing exposure problem.

Auction ends when no new bids offered in a round. If the sum of provisionally winning bids for all licenses in a block exceed the reserve price for block, bids become winners.

Auction 73 outcome: Bidding took 38 days for 262 rounds. 214 bidders participated, 101 bidders won 1090 licenses. AT&T gained nationwide coverage in Block B. Verizon won nationwide coverage block C. Blocks A,B,E made reserve bids, raised around $14B. Block D (nationwide, public safety restrictions) did not make $1.5B reserve bid.

Another large auction is anticipated for 2013.

Some design choices made for Auction 73:

- What spectrum to auction and when (e.g. other blocks of 700MHz band were sold earlier)
- Granularity: Why 1099 licenses? Why groupings?
- Auction format

  Choice of bidding units, minimum bids, reserve prices

Possible design goals:

- Revenue maximization? What time horizon? What revenue? Link payments to generated profits? (Being considered by Indian government.)

- Efficiency (sell to bidders who have highest value for spectrum)?

- Balance interests of incumbents vs. new entrants

- Low complexity of communication and/or computation
  - -for auctioneer
  - -for buyers--including burden of determining good bids

What theory can we call in for this problem?

- Several Nobel Prizes in Economics awarded for mechanism design

  -Hurwicz, Maskin, and Myerson,"for having laid the foundations of mechanism design theory" (last week)

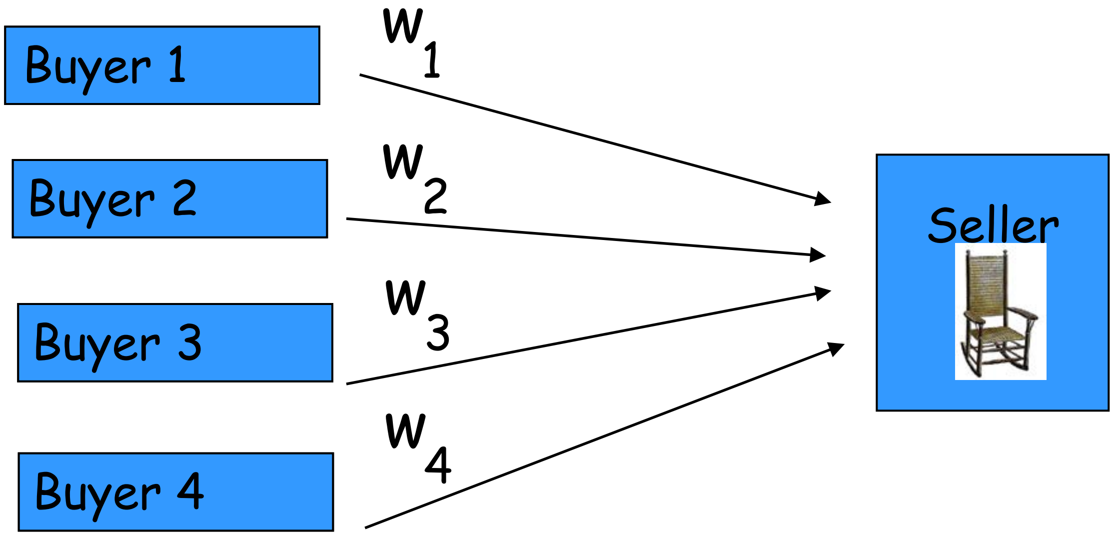
  -Mirrlees and Vickrey, "for fundamental contributions to the economic theory of incentives under asymmetric information" (1996)

  -Harsani,Nash,Selten, "for their pioneering analysis of equilbrium in the theory of noncooperative games"(1994)

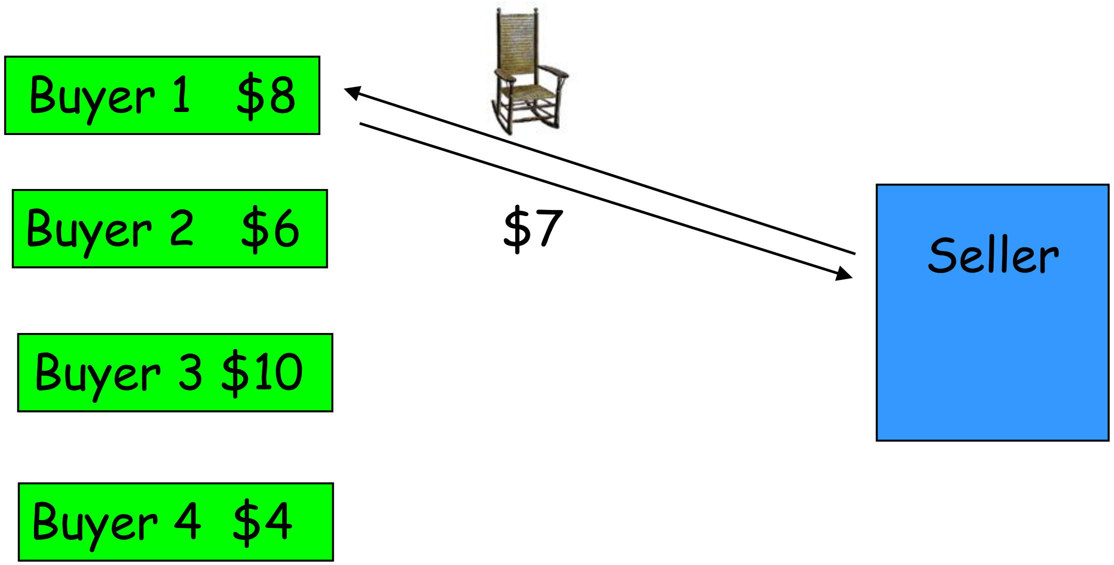
  -other related Nobel's: Samuelson(1970),Debreu(1983)

2. Auction of Single Object

Example: Single object auctions

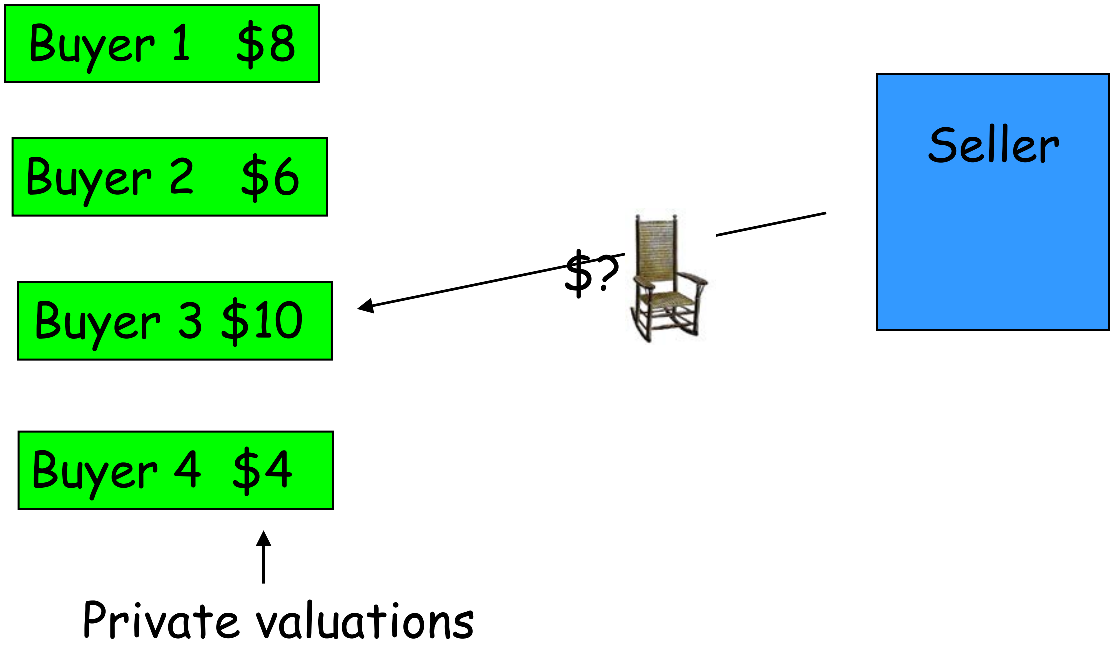


Example: Single object auctions



Note: Given a mechanism, buyers play a game.

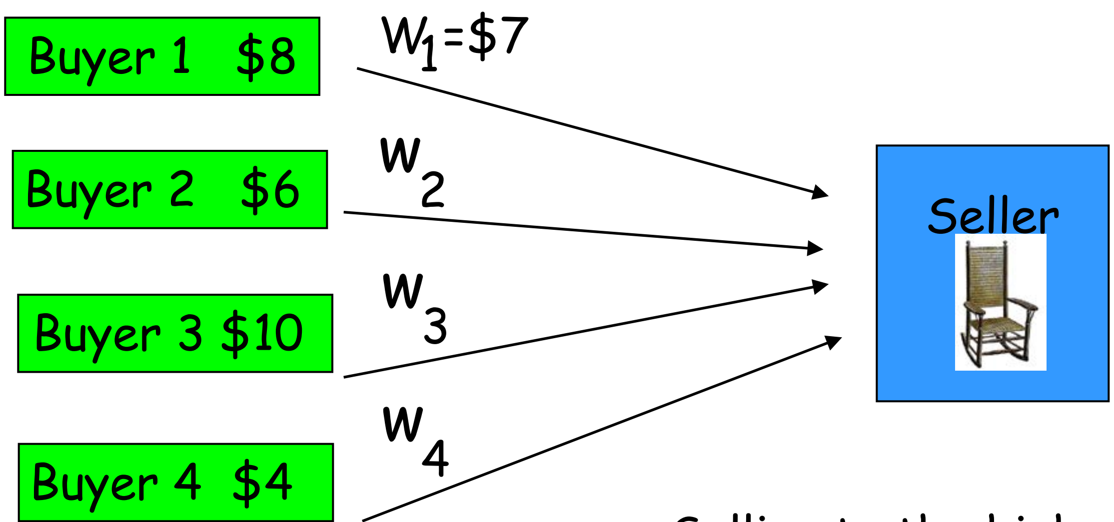
A mechanism is <u>efficient</u> (or socially optimal) if the buyer with the highest valuation gets the object.



A mechanism is <u>incentive compatible</u> if

(1) the bids take values in the same space as the buyer's valuations (states)

(2) each buyer is best off telling the truth, given the other buyers tell the truth



Private valuations

Selling to the highest bidder as bid is <u>not</u> incentive compatible.

A mechanism is incentive compatible if
(1) the bids take values in the same space as the buyer's valuations (states)
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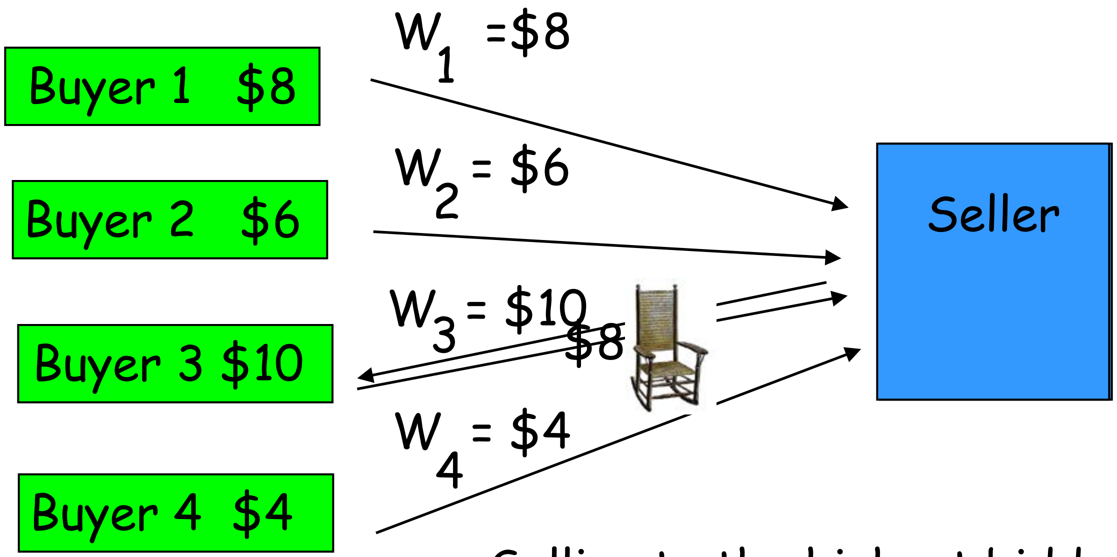
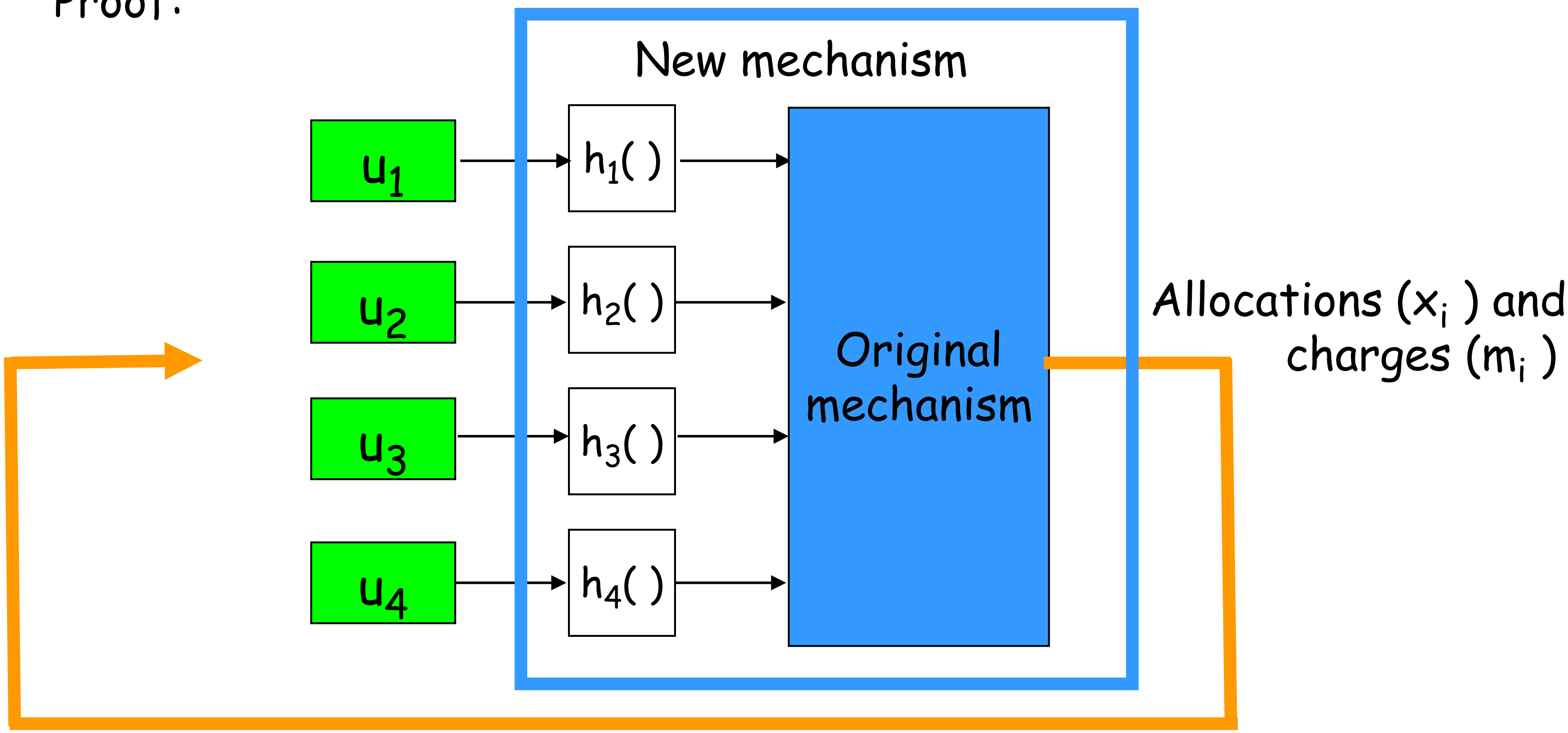


Selling to the highest bidder at the second highest price is incentive compatible and efficient. (Vickrey's second price auction)

Revelation Principle

Revelation principle (Myerson): Given any Nash equilibrium point (NEP) for any mechanism, there is another mechanism for which truth telling is an equivalent NEP (so the other mechanism is IC).

Proof:



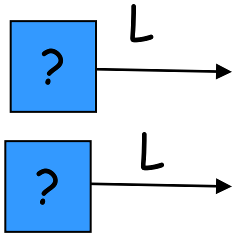
Revelation principle means incentive compatibility can be imposed as a constraint, without loss of generality.

The IC constraint is linear in price and assignment probabilities:

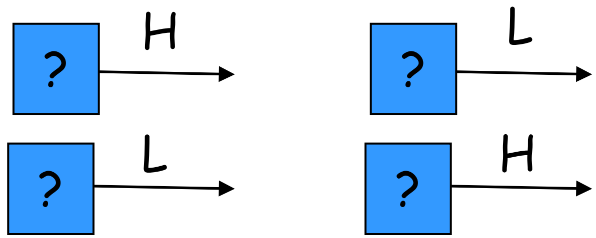
E[reward | bid truth] ≤ E[reward | some other bid]

Example application: Find mechanism to maximize expected revenue for two buyers, with possible valuations L and H, where $0 < L < H$. Equal to H with probability p.

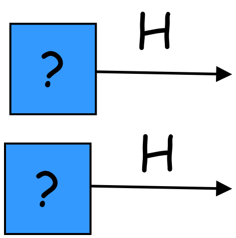
Mechanism maximizing expected revenue:



Sell to bidder selected by fair coin
Price = L



Sell to high bidder
Price = b_H



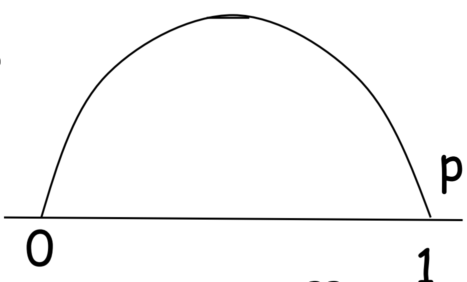
Sell to bidder selected by fair coin
Price = b_H

where b_H is determined by: Given state H: E[reward for truth]=E[reward for lie]

or $$\left(\frac{p}{2}+1-p\right)(H-b_H)=\left(\frac{1-p}{2}\right)(H-L)$$

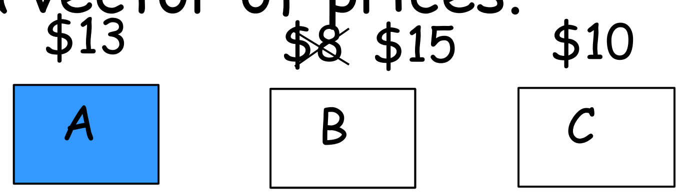
$$b_H=\frac{H+(1-p)L}{2-p}$$

"Information rent" = expected value not extracted by seller
= p(1-p)(H-L)



3. Substitutes property of valuations

Consider the bundle of goods demanded by a bidder as a function of a vector of prices.

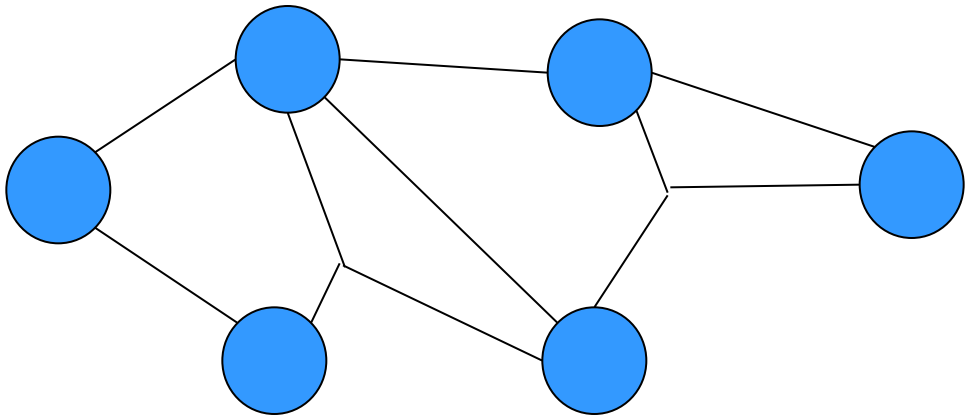


Definition: The bidder valuation function satisfies the *substitutes condition* if when prices are increased on some goods, goods in demanded bundle with no price increase are in a new demand bundle (for the higher prices).

Insures ascending price auction with price taking buyers leads to efficient allocation.

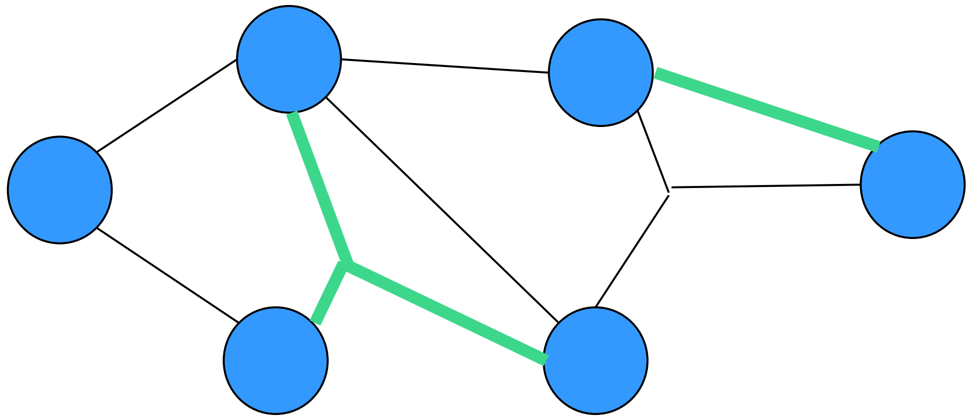
Problem: Can't handle complements.

4. Auctions for single minded buyers



Extreme case of complements
-Nodes of hypergraph are items to be auctioned.
-Edges (bundles) represent buyers.
-For each buyer, both the value and identify of the bundle are private information of the buyer.

Complements/exposure problem is a key motivation for combinatorial auctions.

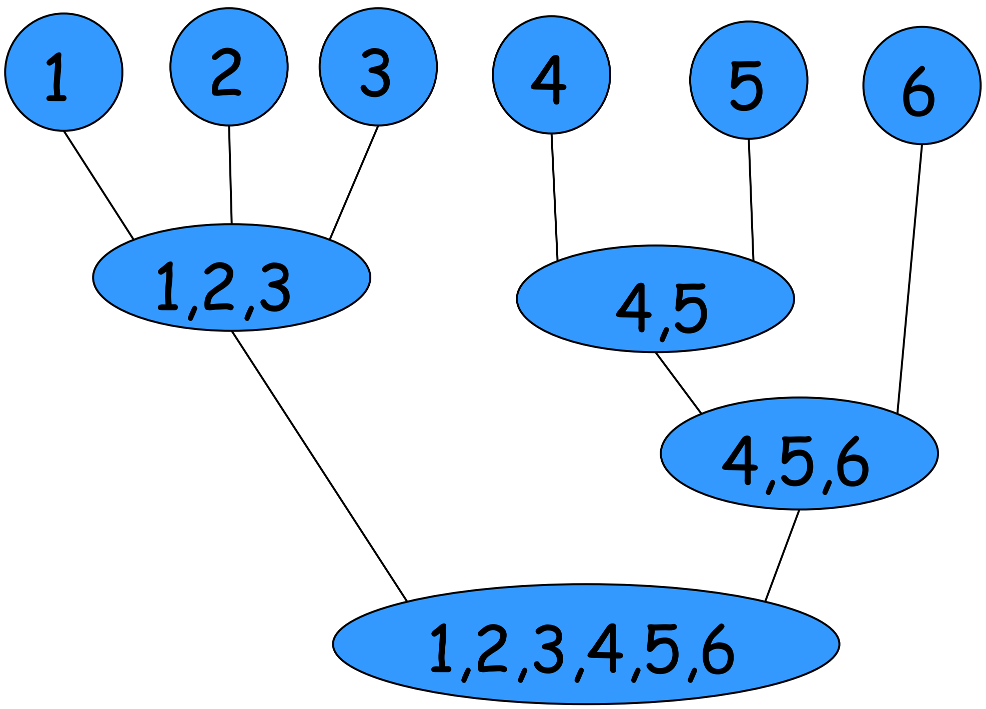


Optimal auction for independent private valuations carries over. Revenue optimal allocation is maximum weight matching, with virtual valuations as weights. Payment of a winning buyer is minimum value needed to win.

-The bundle desired by a bidder can be private information under an ordering condition

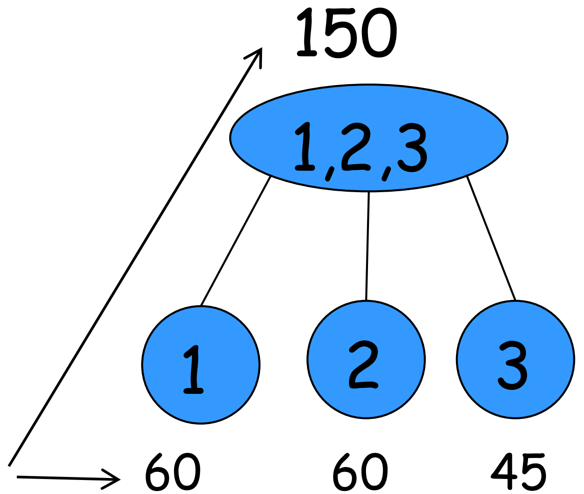
-Efficiency loss for revenue optimal auction with binary valuations is at most 50%. --Abhishek and H, 2009.

Winner determination is simple for bids on bundles decided in advance by auctioneer



But revenue optimal auction depends on knowledge of distribution of valuations.

Focus on payment rules for three small buyers winning against a larger buyer.



	1	2	3
Bids of four buyers:	60	60	45
Vickrey payments:	45	45	30
Projection onto core:	55	55	40

A price vector is in the core if seller cannot make a better deal with a different set of winners.

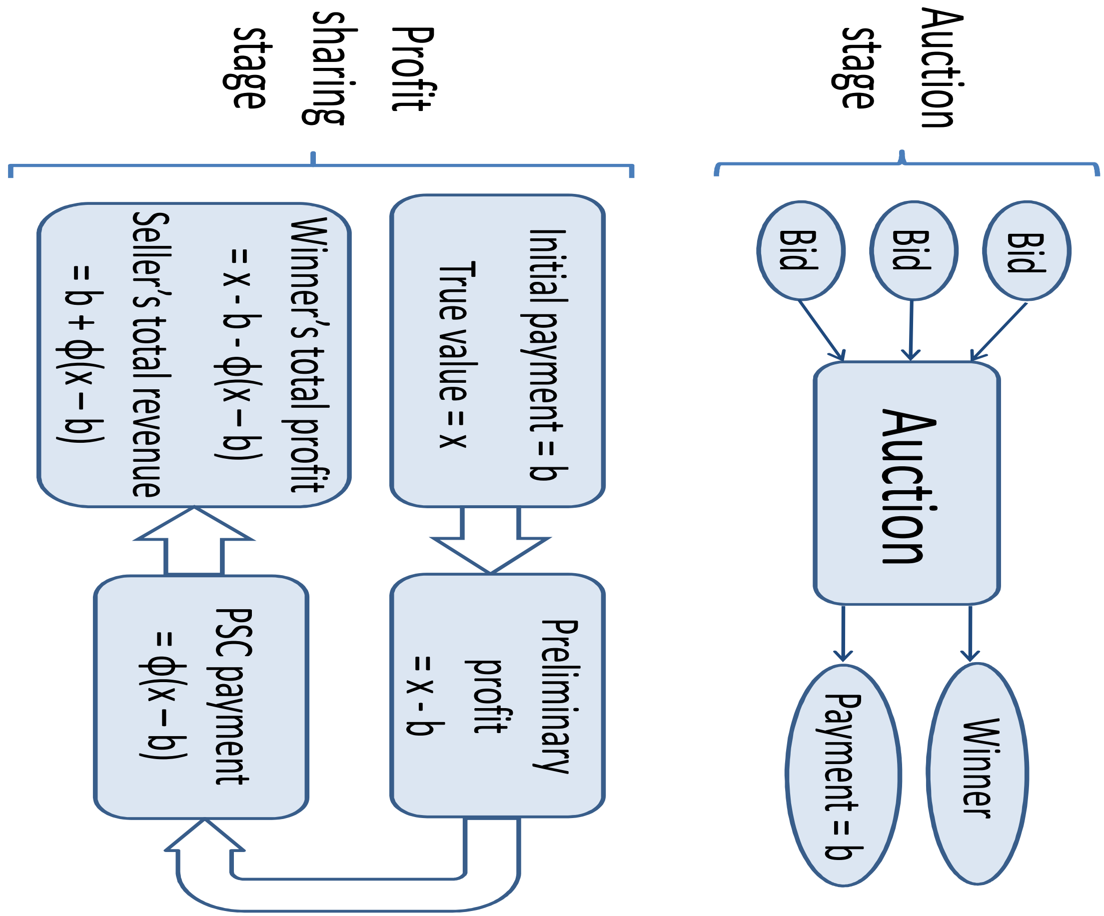
5. Cramton's clock package auction

- Auctioneer names prices; bidder names package
  - Price increased if there is excess demand
  - Process repeated until no excess demand
- Supplementary bids
  - Improve clock bids
  - Bid on other relevant packages
- Optimization to determine assignment/prices

- No exposure problem (package auction)
- Second pricing to encourage truthful bidding
- Activity rule to promote price discovery

For details see Peter Cramton, ***"Spectrum Auction Design,"*** Working Paper, University of Maryland, June 2009.

6. Auction with profit sharing contract



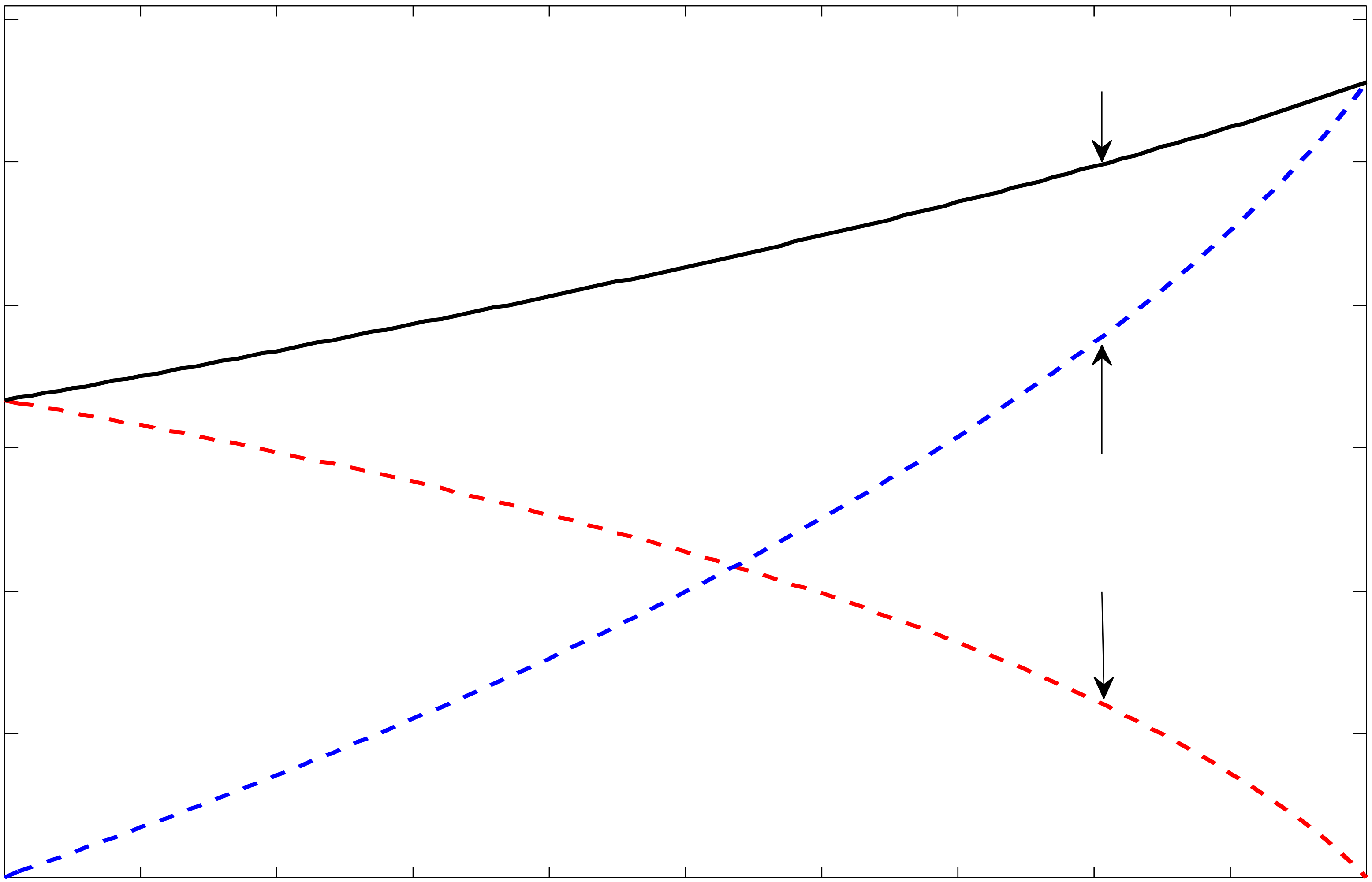
- $\phi(x-b) = \alpha[x-b]_+$ (profit only sharing)
- $\phi(x-b) = \alpha(x-b)$ (profit and loss sharing)

Abhishek, H., and S. R. Williams, 2011

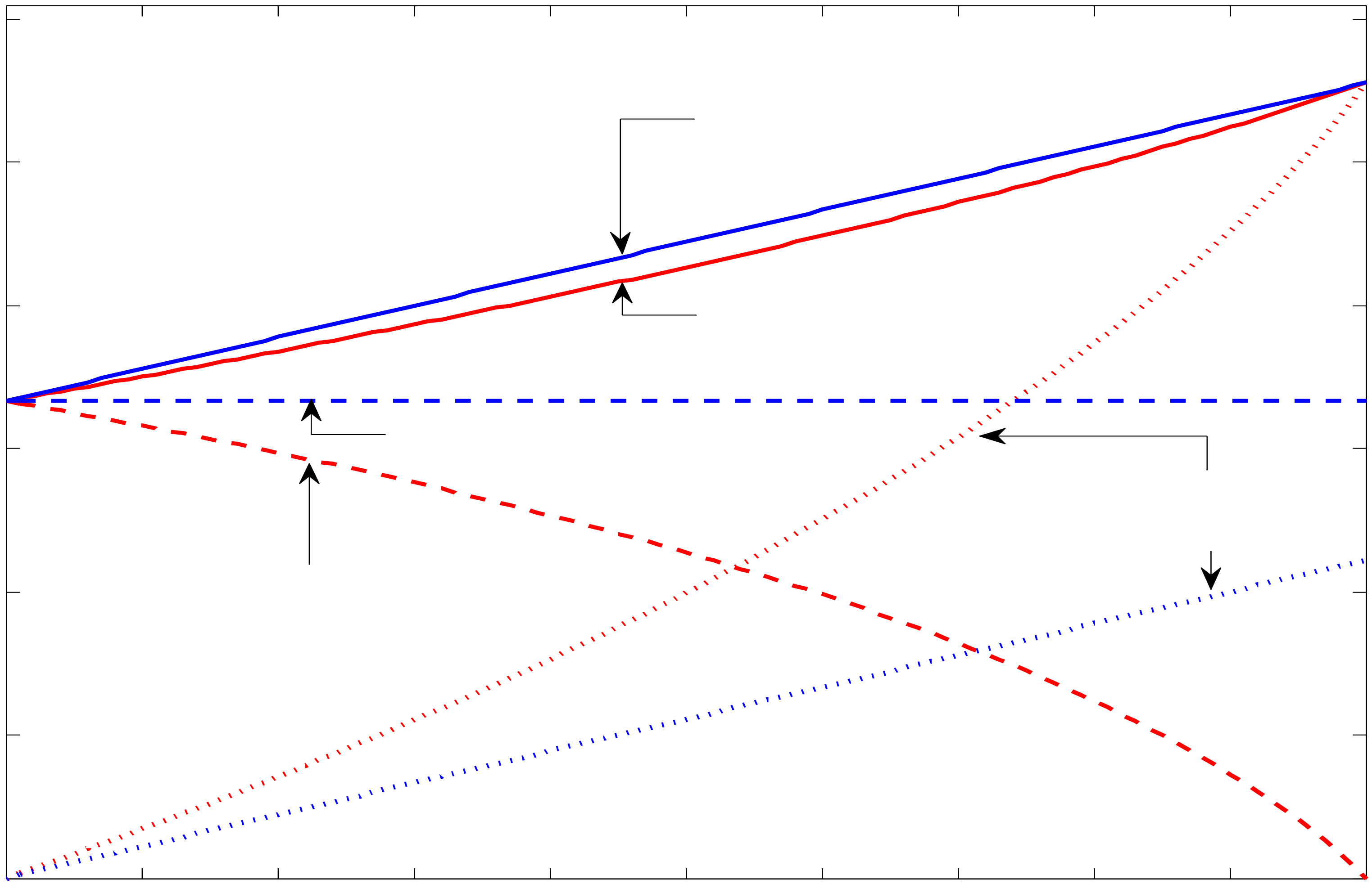
Example of Affiliated Private Signals

- Two risk neutral buyers.
- Values (X_1, X_2), signals (Y_1, Y_2).
- Signals (Y_1, Y_2) i.i.d. unif$(0, 1)$.
- $X_1 \in \{0, 1\}$, $\text{Prob}(X_1 = 1 | Y_1 = y_1, Y_2 = y_2) = (2y_1 + y_2)/3$.
  $X_2 \in \{0, 1\}$, $\text{Prob}(X_2 = 1 | Y_1 = y_1, Y_2 = y_2) = (y_1 + 2y_2)/3$.

Revenue for profit only sharing contract



Revenue comparison: profit only sharing vs. profit and loss sharing



7. Some directions for future work

- Complexity of general winner determination problem remains
- How close to incentive compatible does mechanism remain when Vickrey prices are projected onto core?
- Can affiliation of signals theory be extended from sale of single items to combinatorial auction setting?
- What is the impact of changes in the activity rules?
- What is the impact of changes in which price vector in the core is selected?

Thanks!

Notes and references

1. **US Federal Communications Commission Auctions** Some details on the FCC Auction 73 in 2008 are given in [17, 18]. The auction method resembles that described in [9, 23].

2. **Efficient or revenue optimal auctions of a single item** Vickrey [26] first described second price auctions and Myerson [25] first described the revelation principle and calculation of optimal mechanisms for Bayes-Nash equilibrium. Elkind's [16] translated Myerson's results to a setting with discrete valuations.

3. **Substitute valuations in equilibrium economics** Walras' theory of equilibrium in an exchange economy dates back at least to [27], including the theory of tâtonnement processes (or groping process). Arrow and Debreu and McKenzie developed a general theory of equilibrium in the 1950's. See, for example, [5] and [14]. The main convergence results for tâtonnement processes

are derived assuming the valuation functions of the individuals satisfy a substitutes condition.

For discrete goods, Kelso and Crawford [21] gave an ascending price adjustment process, which under a substitutes and other conditions, converges to an efficient allocation. See [6, 7, 19, 22] for ascending price auctions for discrete goods, with valuation functions that are assumed to satisfy a substitutes condition. Milgrom and Strulovic [22] describe a min-norm pseudo-gradient algorithm to computing the dual optimal price vector, which they call the pseudo-equilibrium price vector. Their algorithm works for mulitunit auctions under a strong substitutes condition.

It is shown in [20] that the dimension of the set of valuation functions on n objects, as a subset of 2^n, has dimension greater than $2^{(1-\epsilon)n}$ for sufficiently large n, for any $\epsilon > 0$. However, substitute valuation functions appearing in practice are rarely outside the n^2 dimensional subclass of valuation functions arising as the values of optimal matchings in a bipartite graph.

4. **Auctions for single-minded buyers and efficiency of revenue optimal auctions** [1] derived tight bounds on the loss of efficacy implied by revenue maximization for buyers with independent, one-dimensional valuations, and [2] showed that the Myerson framework extends to the two-dimensional setting in which each buyer has both a private single-minded preference for a particular bundle and a value for that bundle.

5. **Package clock auctions and projection onto core** Package clock auctions, as described by P. Camton [10], were used in two recent UK auctions. Day and Milgrom [13] describes motivations and some properties of core-selecting payment rules while Day and Crampton [12] discusses using the particular price vector in the core that is closest in the L^2 sense to the Vickrey price vector. Preliminary work on alternative selections from the core are analyzed in [8]. Cramton [11] describes a proposal for a US spectrum white-space auction expected around 2013.

6. **Auctions with profit sharing contracts**

Auctions with profit sharing contracts were considered by [3]. Signals of buyers are affiliated as in Milgrom and Weber [24]. The mathematics is related to a partial ordering on one-dimensional families of securities defined by an upcrossing condition [4, 15].

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