A Linear-Complexity Resource Allocation Method for Heterogeneous Multiuser OFDM Downlink

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***Abstract*—In this paper, we consider the dynamic power and rate allocation for the heterogeneous transmission over multiuser orthogonal frequency division multiplexing (OFDM) downlink, where users may require real time or non-real time transmission. The data rate for the real time transmission is lower bounded, while the total transmission power for both transmissions is limited to a fixed amount. Solutions to this problem must be computationally efficient in order to adapt to the fast-varying channels in practice. To accelerate the resource allocation for the considered scenario, efficient approaches are given to update the power or rate variation while changing subcarrier assignments. By iteratively using these approaches, a resource allocation method is proposed to achieve a good balance between the performance and the complexity. Its complexity is linearly increasing in the number of subcarriers and the number of users. Simulation results demonstrate that our method has small performance loss compared to the dual optimum and achieves much better performance compared to previous works.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) [1] has been proposed to efficiently combat the inter-symbol interference effect in the high-speed wireless data applications due to its relatively simple receiver. In OFDM, the wide transmission band with frequency selective fading is equally divided into subcarriers. If the number of subcarriers is large enough, each subcarrier is subject to flat fading. This allows that different powers and rates can be allocated to different subcarriers depending on channel characteristics.

In multiuser OFDM downlink, spatial diversity among users in different locations is employed to enhance the bandwidth efficiency and system performance. Users in multiuser OFDM downlink can be classified into two groups according to their different requirements. One is composed of the margin-adaptive (MA) users. Each of them requires a fixed transmission data rate and a certain bit error rate (BER). The other consists of the rate-adaptive (RA) users only with BER requirements, while the total transmission power is limited. Optimal, dual optimal, near optimal and suboptimal solutions given in [2]–[5] haven been suggested for only considering the MA users in downlink. The resource allocation only for the RA users has been studied in [6], [7], where the optimal solution may be achieved with linear complexity.

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However, both groups of users may appear simultaneously in the multiuser OFDM downlink. The resource allocation under this condition is relatively less studied. The optimal solution to this problem can be obtained with the multilevel water-filling by using a similar idea as given in [2]. Its complexity is an exponentially increasing function in the number of subcarriers. The dual optimum to this problem can be acquired by the dual method [8]–[10]. Its complexity is sup-linearly increasing either in the number of subcarriers or in the number of users, which ought to be very large in future communication systems. Therefore, these methods cannot be implemented in large systems. The heuristic method suggested in [11] gives a suboptimal solution, where two consecutive steps are performed. First, the power and rate allocation is performed only for the MA users. Then, the rest resource is assigned to the RA users. By using a similar procedure, [12] gives the suboptimal solution by performing a greedy search over subcarriers.

In this paper, the resource allocation problem for the heterogeneous transmission over multiuser OFDM downlink is studied, where the transmission to the MA and RA users is jointly considered. A linear-complexity method is proposed to solve this problem with small performance loss. The remainder of this paper is organized as follows. The problem is expressed in mathematical terms in Section II. In Section III, we analyze the variations of the consumed power for an MA user and the achieved rate for the RA users, when the subcarrier assignments are changed. Efficient updating approaches are provided. To use these approaches while achieving high performance efficiency, a heuristic method with low complexity is designed in Section IV. Simulation results are shown in Section V. Finally, the content of this paper is concluded.

II. Preliminaries

Consider the downlink of a multiuser OFDM system with N subcarriers. The transmission to different users experiences independent frequency selective fading. It is assumed that perfect channel knowledge is available at the base station and all users. The uncoded modulation scheme of quadrature amplitude modulation (QAM) is employed. In a specific period of time, the base station provides transmission to K MA users with individual rate requirements R_k and $\text{BER}_k^{(\text{MA})}$. Meanwhile, it sends data to L RA users only with BER demand $\text{BER}_l^{(\text{RA})}$. We denote by $\mathcal{K}$ and $\mathcal{L}$ the sets of the K MA users and the L RA users, respectively. Let $g_{k,n}^{(\text{MA})}$ and $g_{l,n}^{(\text{RA})}$ denote the channel gain-to-noise ratio (CNR) of subcarrier n multiplied with $\left[-1.5/\ln(5\text{BER}_k^{(\text{MA})})\right]$ and $\left[-1.5/\ln(5\text{BER}_l^{(\text{RA})})\right]$ for the MA user k and the RA user l [13], respectively. The power allocated to subcarrier n for these MA and RA users are indicated by $p_{k,n}^{(\text{MA})}$ and $p_{l,n}^{(\text{RA})}$. Accordingly, the allocated rates [1] can be obtained as

$$\begin{aligned} r_{k,n}^{(\text{MA})} &= \log_2(1 + p_{k,n}^{(\text{MA})} g_{k,n}^{(\text{MA})}) \text{ and} \\ r_{l,n}^{(\text{RA})} &= \log_2(1 + p_{l,n}^{(\text{RA})} g_{l,n}^{(\text{RA})}). \end{aligned}$$

Certainly, each subcarrier is not allowed to be shared by different users at any time.

We aim at maximizing the total transmission rate for the RA users while satisfying the data rate and BER requirements for the MA users. The optimization problem can be expressed as

$$\begin{aligned} \text{maximize} \quad & \sum_{l=1}^{L} \sum_{n=1}^{N} r_{l,n}^{(\text{RA})} && (1) \\ \text{subject to} \quad & \sum_{n=1}^{N} r_{k,n}^{(\text{MA})} \geq R_k, && \forall k \in \mathcal{K}, \\ & P^{(\text{RA})} + \sum_{k=1}^{K} P_k^{(\text{MA})} \leq P^{(\text{tot})}, \\ & p_{l,n}^{(\text{RA})} \geq 0, && \forall l \in \mathcal{L}, \forall n, \\ & p_{k,n}^{(\text{MA})} \geq 0, && \forall k \in \mathcal{K}, \forall n, \\ & \sum_{n=1}^{N} p_{l,n}^{(\text{RA})} p_{m,n}^{(\text{RA})} = 0, && \forall l, m \in \mathcal{L}, l \neq m, \\ & \sum_{n=1}^{N} p_{k,n}^{(\text{MA})} p_{m,n}^{(\text{MA})} = 0, && \forall k, m \in \mathcal{K}, k \neq m, \\ & \sum_{n=1}^{N} p_{k,n}^{(\text{RA})} p_{l,n}^{(\text{MA})} = 0, && \forall k \in \mathcal{K}, \forall l \in \mathcal{L}. \end{aligned}$$

The transmission power allocated to MA user k is $P_k^{(\text{MA})} = \sum_{n=1}^{N} p_{k,n}^{(\text{MA})}$. The transmission power for the RA users is $P^{(\text{RA})} = \sum_{l=1}^{L} \sum_{n=1}^{N} p_{l,n}^{(\text{RA})}$. The total transmission power is limited to $P^{(\text{tot})}$. To allow for theoretical analysis, continuous rates are considered throughout this paper. The subcarrier assignment of MA user k is denoted by the set $\mathcal{S}_k = \{n \mid r_k[n] > 0\}$ with cardinality s_k. Set $\mathcal{S}_{K+1}$ consists of the s_{K+1} subcarriers assigned to the RA users.

III. Variation of Power consumption and Rate Achievement by Varying a Subcarrier Assignment

From [6], if a subcarrier is assigned to the RA users, it must be assigned to the RA user who has the largest CNR on this subcarrier. Hence, the RA users can be treated as one RA user with CNRs

$$g_n^{(\text{RA})} = \max_{l \in \mathcal{L}} g_{l,n}^{(\text{RA})}, \ \forall n.$$

Water-filling [1] can be used to obtain the optimal power and rate allocation, once the transmission power for the RA users and the subcarrier assignments for all users are determined.

After a subcarrier is excluded from or added to the subcarrier assignment for an MA user or the RA users, the transmission power for the MA users or the achieved transmission rate for the RA users may vary. Such variations can be derived with water-filling, whose complexity is linearly increasing in the cardinality of the subcarrier assignment, see [14]–[16]. Therefore, it is very computationally complex to iteratively use water-filing in multiuser resource allocation, see [17]–[19]. Under certain conditions, the power or rate variation can be calculated much more efficiently.

Given subcarriers assignment $\mathcal{S}_k$ for MA user k, it is assumed that the positive rate and power allocated to any subcarrier $n \in \mathcal{S}_k$ are

$$\begin{aligned} r_{k,n}^{(\text{MA})} &= \log_2(\lambda_k g_{k,n}^{(\text{MA})}) \text{ and} \\ p_{k,n}^{(\text{MA})} &= \lambda_k - \frac{1}{g_{k,n}^{(\text{MA})}}. \end{aligned}$$

The water level λ_k can be determined by

$$\lambda_k = 2^{\frac{R_k}{s_k}} \Big(\prod_{n \in \mathcal{S}_k} \frac{1}{g_{k,n}^{(\text{MA})}}\Big)^{\frac{1}{s_k}}. \tag{2}$$

Provided the subcarrier assignment $\mathcal{S}_{K+1}$ and the transmission power $P^{(\text{RA})}$ for the RA users, the allocated rate and power for the RA users are assumed to be positive, as

$$\begin{aligned} r_n^{(\text{RA})} &= \log_2(\beta g_n^{(\text{RA})}) \text{ and} \\ p_n^{(\text{RA})} &= \beta - \frac{1}{g_n^{(\text{RA})}}, \end{aligned}$$

where the water level is

$$\beta = \frac{P^{(\text{RA})} + \sum_{n \in \mathcal{S}_{K+1}} 1/g_n^{(\text{RA})}}{s_{K+1}}. \tag{3}$$

After excluding subcarrier m from $\mathcal{S}_k$ for MA user k, the water level increases to

$$\lambda_k^{(\text{r})}(m) = \lambda_k (\lambda_k g_{k,m}^{(\text{MA})})^{\frac{1}{s_k - 1}},$$

where the upper index (r) indicates the operation of removing subcarrier m from the subcarrier assignment. The resulting power increment is

$$\Delta P_k^{(\text{r})}(m) = (s_k - 1)(\lambda_k^{(\text{r})}(m) - \lambda_k) - (\lambda_k - 1/g_{k,m}^{(\text{MA})}).$$

It illustrates that the power increments on the remaining $s_k - 1$ subcarriers are the same and equal to the increment of the water level, seen from the first term. The second term is the power decrement by not allocating power to subcarrier m. The sum of the power increments and the power decrement must be positive due to the increasing water level.

If subcarrier m is excluded from the subcarrier assignment $\mathcal{S}_{K+1}$ for the RA users, the water level becomes

$$\beta^{(\text{r})}(m) = \frac{s_{K+1}\beta - 1/g_m^{(\text{RA})}}{s_{K+1} - 1}.$$

The induced rate decrement is

$$\Delta R^{(r)}(m) = (s_{K+1}-1)\log_2(\beta^{(r)}(m)/\beta) - \log_2(\beta g_m^{(RA)}),$$

where the rate increments on the remaining $s_{K+1}-1$ subcarriers are the same, and the excluded subcarrier does not carry any bit for the RA users.

Similarly, after adding subcarrier m with $1/g_{k,m}^{(MA)} < \lambda_k$ to $\mathcal{S}_k$ of MA user k, the water level decreases to

$$\lambda_k^{(a)}(m) = \lambda_k(\lambda_k g_{k,m}^{(MA)})^{-\frac{1}{s_k+1}},$$

and the power decrement is

$$\Delta P_k^{(a)}(m) = (s_k+1)(\lambda_k^{(a)}(m) - \lambda_k) + (\lambda_k - 1/g_{k,m}^{(MA)}),$$

where (a) indicates the operation of adding subcarrier m to the subcarrier assignment. The power increment $\lambda_k - 1/g_{k,m}^{(MA)}$ is the allocated power to subcarrier m given the previous water level. The power decrement on each subcarrier in the new subcarrier assignment is the decrement of the water level. The overall power variation is the sum of the power increment and decrements. This updating approach may provide non-optimal solutions, when the CNR of subcarrier m is very large and the rates allocated to some other subcarriers become negative.

When subcarrier m with $1/g_{k,m}^{(RA)} < \beta$ is added to $\mathcal{S}_{K+1}$ for the RA users, the water level reduces to

$$\beta^{(a)}(m) = \frac{s_{K+1}\beta + 1/g_m^{(RA)}}{s_{K+1}+1}.$$

Then, the rate increment is

$$\Delta R^{(a)}(m) = (s_{K+1}+1)\log_2(\beta^{(a)}(m)/\beta) + \log_2(\beta g_m^{(RA)}),$$

where the rate $\log_2(\beta g_m^{(RA)})$ is allocated on the added subcarrier, and the rate decrements on the subcarriers in $\mathcal{S}_{K+1}$ are the same. This updating approach may also be non-optimal due to the reduction of the water level.

If the transmission power for the RA users varies by $\Delta P^{(RA)}$, the new water level is

$$\beta^{(p)}(\Delta P^{(RA)}, \beta) = \beta + \Delta P^{(RA)}/s_{K+1},$$

where (p) indicates the operation of changing the power for the RA users by $\Delta P^{(RA)}$. The induced rate variation on each subcarrier is the same. The variation sum

$$\Delta R^{(p)}(\Delta P^{(RA)}, \beta) = s_{K+1}\log_2(\beta^{(p)}/\beta)$$

may be non-optimal, when $\Delta P^{(RA)}$ is a very small negative number resulting in negative rates on some subcarriers.

It can be seen that the complexity is very low to derive the power and rate variations and the new water levels while changing subcarrier assignments. We use only one exponential operation to obtain the pair of $(\lambda_k^{(r)}(m), \Delta P_k^{(r)}(m))$ or the pair of $(\lambda_k^{(a)}(m), \Delta P_k^{(a)}(m))$. Only at most two logarithms operations are needed to obtain the pair $(\beta^{(r)}(m), \Delta R^{(r)}(m))$, the pair $(\beta^{(a)}(m), \Delta R^{(a)}(m))$ or the pair $(\beta^{(p)}(\Delta P^{(RA)}, \beta), \Delta R^{(p)}(\Delta P^{(RA)}, \beta))$. The other operations are just additions and subtractions. However, solutions may be not optimal, while the constraint of the non-negative allocated power in (1) is not met. Hence, a good initialization for the subcarrier assignments is necessary in order to use the efficient updating approaches while keeping the performance loss small. We will discuss this in the following.

Algorithm 1 Cardinality Evaluation

$d_k \leftarrow 1,\ \forall k \in \mathcal{K} \cup \{K+1\}$
$g_k^{(MA)} \leftarrow (\prod_{n=1}^N g_{k,n}^{(MA)})^{\frac{1}{N}},\ \forall k \in \mathcal{K}$
$g^{(RA)} \leftarrow \frac{1}{N}\sum_{n=1}^N g_n^{(RA)}$
repeat
  $P_k^{(MA)} \leftarrow d_k/g_k^{(MA)}(2^{\frac{R_k}{d_k}} - 1), \forall k \in \mathcal{K}$
  $\Delta P_k^{(MA)} \leftarrow P_k^{(MA)} - (d_k+1)/g_k^{(MA)}(2^{\frac{R_k}{d_k+1}} - 1), \forall k \in \mathcal{K}$
  $\tilde{k} \leftarrow \mathrm{argmax}_{k\in\mathcal{K}}\, \Delta P_k^{(MA)}$
  if $\sum_{k=1}^K P_k^{(MA)} > P^{(tot)}$ **then**
    $d_{\tilde{k}} \leftarrow d_{\tilde{k}} + 1$
  else
    $R^{(MA)} \leftarrow d_{K+1}\log_2\left(1 + \frac{P^{(tot)} - \sum_{k=1}^K P_k^{(MA)} + \Delta P_{\tilde{k}}^{(MA)}}{d_{K+1}g^{(RA)}}\right)$
    $R^{(RA)} \leftarrow (d_{K+1}+1)\log_2\left(1 + \frac{P^{(tot)} - \sum_{k=1}^K P_k^{(MA)}}{(d_{K+1}+1)g^{(RA)}}\right)$
    if $R^{(MA)} > R^{(RA)}$ **then**
      $d_{\tilde{k}} \leftarrow d_{\tilde{k}} + 1$
    else
      $d_{K+1} \leftarrow d_{K+1} + 1$
    end if
  end if
until $\sum_{k=1}^{K+1} d_k = N$

IV. Heuristic Power and Rate Allocation for Heterogeneous Transmission

If the sum of the transmission power for the MA users is not larger than $P^{(tot)}$ without considering the RA users, problem (1) is solvable. Otherwise, upper layers or other protocols must adjust the BER or rate requirements, but this is beyond the scope of this work. In the previous work [11], first, the power and subcarriers are assigned to the MA users, then, the remaining resource is assigned to the RA users. This factorization to problem (1) cannot reduce the solving complexity significantly and worsens the non-optimal solution. Alternatively, we may use the dual method [20] to solve problem (1) with a very small duality gap [3], [8], [10], while the ellipsoid method is applied to find the $K+1$ dual optimal Lagrange multipliers. However, the complexity is $\mathcal{O}((K+1)^3N)$ [21], which makes the solution not implementable for the case of many MA users.

In this section, we introduce a low-complexity method to solve problem (1) consisting of three consecutive steps, while the efficient approaches explained in the previous section are utilized in the third step. As earlier mentioned, the subcarriers with negative power allocated must be very few during this utilization to keep the performance loss small. Therefore, the first two steps provides a good starting point of simply distributing subcarriers to the MA and RA users.

Algorithm 2 Initialization of Subcarrier Assignments

$\mathcal{N} \leftarrow \{1, \ldots, N\}$
$\mathcal{S}_k \leftarrow \emptyset,\ \forall k \in \mathcal{K} \cup \{K+1\}$
$d \leftarrow (\prod_{k=1}^{K+1} d_k)^{\frac{1}{K+1}}$
$D_k \leftarrow \lceil d_k/d \rceil,\ \forall k \in \mathcal{K} \cup \{K+1\}$
repeat
  for each $k \in \mathcal{K} \cup \{K+1\}$ **do**
    if $|\mathcal{S}_k| < d_k$ **then**
      $\hat{D}_k \leftarrow \min(d_k - |\mathcal{S}_k|, D_k)$
      $\mathcal{T} \leftarrow \{\hat{D}_k \text{ subcarriers with the largest CNRs}, n \in \mathcal{N}\}$
      $\mathcal{S}_k \leftarrow \mathcal{S}_k \cup \mathcal{T}$
      $\mathcal{N} \leftarrow \mathcal{N} \setminus \mathcal{T}$
    end if
  end for
until $\mathcal{N} = \emptyset$
$(P_k^{(\mathrm{MA})}, \lambda_k, \mathcal{S}_k) \leftarrow \mathrm{SUWF}^{(\mathrm{MA})}(\mathcal{S}_k, R_k),\ \forall k \in \mathcal{K}$
$P^{(\mathrm{RA})} \leftarrow P^{(\mathrm{tot})} - \sum_{k=1}^{K} P_k^{(\mathrm{MA})}$
$(\beta, \mathcal{S}_{K+1}) \leftarrow \mathrm{SUWF}^{(\mathrm{RA})}(\mathcal{S}_{K+1}, P^{(\mathrm{RA})})$

A. Cardinality evaluation

Algorithm 1 returns the evaluated cardinality of each subcarrier assignment d_k, which is set to one at the beginning. To avoid high complexity, all subcarriers for each user are assumed to have the same CNR equal to the the geometric average CNR over subcarriers $g_k^{(\mathrm{MA})}$ instead of the arithmetic average in [22] due to the geometric mean in (2). The CNRs over subcarriers for the RA users are arithmetically averaged to $g^{(\mathrm{RA})}$. On one side, the transmission power for the RA users would increase, if the cardinality of the subcarrier assignment for an MA user increases. On the other side, the rate achievement for the RA users would be enhanced, when more subcarriers were assigned to the RA users.

In each iteration, if the transmission power for the MA users $\sum_{k=1}^{K} P_k^{(\mathrm{MA})}$ is larger than $P^{(\mathrm{tot})}$, we increase the cardinality of the subcarrier assignment only for the MA user, who has the largest power reduction. Otherwise, we increase the cardinality of the subcarrier assignment for the MA user or the RA users inducing the largest improvement on the output rate for the RA users. This iteration finishes, when the sum of cardinalities is equal to N. Since every cardinality must be evaluated in each iteration, the complexity of Algorithm 1 is $\mathcal{O}(KN)$.

B. Initialization of subcarrier assignments

According to the evaluated cardinalities from Algorithm 1, subcarriers are simply distributed to users in Algorithm 2. This can reduce the performance loss by using the efficient approaches from the previous section after adding a subcarrier to a subcarrier assignment. In each iteration, each MA user k obtains D_k subcarriers, and the RA users get D_{K+1} subcarriers with the largest CNRs in $\mathcal{N}$, which are determined with the geometric mean of the evaluated cardinalities. This can be implemented by the order statistic algorithm [23] with complexity $\mathcal{O}(|\mathcal{N}|)$. It may happen that the remaining number of subcarriers needed by user k, $d_k - |\mathcal{S}_k|$, is smaller than D_k. Only $d_k - |\mathcal{S}_k|$ subcarriers are selected from $\mathcal{N}$ for this case. The iteration stops, when $\mathcal{N}$ is empty. At last, the water levels and subcarrier assignments are determined by the single-user water-filling $\mathrm{SUWF}^{(\mathrm{MA})}$ for each MA user and $\mathrm{SUWF}^{(\mathrm{RA})}$ for the RA users, see [1]. Due to the low complexity of the order statistic algorithm and the simple iteration without any calculation, the complexity of this step is $\mathcal{O}(KN)$.

Algorithm 3 Successive Subcarrier Adjustment

for each subcarrier m **do**
  $\mathcal{B} \leftarrow \{k \in \mathcal{K} \mid m \in \mathcal{S}_k\}$
  $\Delta P^{(\mathrm{r})}(m) \leftarrow 0$
  if $|\mathcal{S}_k| = 1, k \in \mathcal{B}$ **then**
    $\Delta P^{(\mathrm{r})}(m) \leftarrow \infty$
  else if $\mathcal{B} \neq \emptyset$ **then**
    $\Delta P^{(\mathrm{r})}(m) \leftarrow \Delta P_k^{(\mathrm{r})}(m), k \in \mathcal{B}$
  end if
  $\mathcal{U} \leftarrow \{k \in \mathcal{K} \mid \lambda_k g_{k,m}^{(\mathrm{MA})} > 1\} \setminus \mathcal{B}$
  $\Delta P_k^{(\mathrm{MA})}(m) \leftarrow \Delta P_k^{(\mathrm{a})}(m) + \Delta P^{(\mathrm{r})}(m),\ \forall k \in \mathcal{U}$
  $\hat{k} \leftarrow \mathrm{argmin}_{k \in \mathcal{U}}\, \Delta P_k^{(\mathrm{MA})}(m)$
  if $\sum_{k=1}^{K} P_k^{(\mathrm{MA})} > P^{(\mathrm{tot})}$ **then**
    if $\Delta P_{\hat{k}}^{(\mathrm{MA})}(m) < 0$ **then**
      $\lambda_k \leftarrow \lambda_k^{(\mathrm{r})}(m), k \in \mathcal{B}$
      $\mathcal{S}_k \leftarrow \mathcal{S}_k \setminus \{m\}, k \in \mathcal{B}$
      $\lambda_{\hat{k}} \leftarrow \lambda_{\hat{k}}^{(\mathrm{a})}(m)$
      $\mathcal{S}_{\hat{k}} \leftarrow \mathcal{S}_{\hat{k}} \cup \{m\}$
    end if
  else
    if $m \notin \mathcal{S}_{K+1}$ **then**
      if $\Delta R^{(\mathrm{a})}(m) + \Delta R^{(\mathrm{p})}(-\Delta P^{(\mathrm{r})}(m), \beta^{(\mathrm{a})}(m)) > \left(\Delta R^{(\mathrm{p})}(-\Delta P_{\hat{k}}^{(\mathrm{MA})}(m), \beta)\right)^{+}$ **then**
        $\mathcal{S}_{K+1} \leftarrow \mathcal{S}_{K+1} \cup \{m\}$
        $\beta \leftarrow \beta^{(\mathrm{p})}(-\Delta P^{(\mathrm{r})}(m), \beta^{(\mathrm{a})}(m))$
        $\lambda_k \leftarrow \lambda_k^{(\mathrm{r})}(m), k \in \mathcal{B}$
        $\mathcal{S}_k \leftarrow \mathcal{S}_k \setminus \{m\}, k \in \mathcal{B}$
      else if $\left(\Delta R^{(\mathrm{a})}(m) + \Delta R^{(\mathrm{p})}(-\Delta P^{(\mathrm{r})}(m), \beta^{(\mathrm{a})}(m))\right)^{+} < \Delta R^{(\mathrm{p})}(-\Delta P_{\hat{k}}^{(\mathrm{MA})}(m), \beta)$ **then**
        $\beta \leftarrow \beta^{(\mathrm{p})}(-\Delta P_{\hat{k}}^{(\mathrm{MA})}(m), \beta)$
        $\lambda_k \leftarrow \lambda_k^{(\mathrm{r})}(m), k \in \mathcal{B}$
        $\mathcal{S}_k \leftarrow \mathcal{S}_k \setminus \{m\}, k \in \mathcal{B}$
        $\lambda_{\hat{k}} \leftarrow \lambda_{\hat{k}}^{(\mathrm{a})}(m)$
        $\mathcal{S}_{\hat{k}} \leftarrow \mathcal{S}_{\hat{k}} \cup \{m\}$
      end if
    else if $\Delta R^{(\mathrm{r})}(m) + \Delta R^{(\mathrm{p})}(-\Delta P_{\hat{k}}^{(\mathrm{a})}(m), \beta^{(\mathrm{r})}(m)) > 0$ **then**
      $\mathcal{S}_{K+1} \leftarrow \mathcal{S}_{K+1} \setminus \{m\}$
      $\beta \leftarrow \beta^{(\mathrm{p})}(-\Delta P_{\hat{k}}^{(\mathrm{a})}(m), \beta^{(\mathrm{r})}(m))$
      $\lambda_{\hat{k}} \leftarrow \lambda_{\hat{k}}^{(\mathrm{a})}(m)$
      $\mathcal{S}_{\hat{k}} \leftarrow \mathcal{S}_{\hat{k}} \cup \{m\}$
    end if
  end if
end for

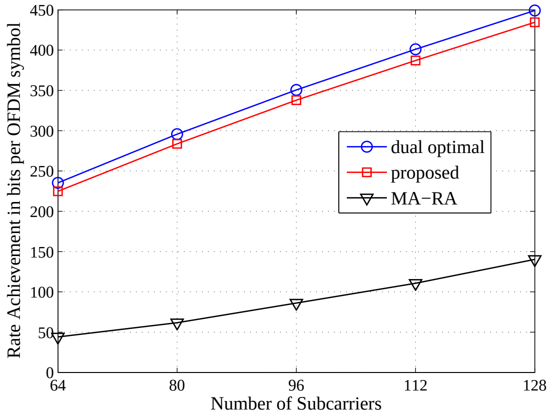

Fig. 1. Rate achievement vs. number of subcarriers.

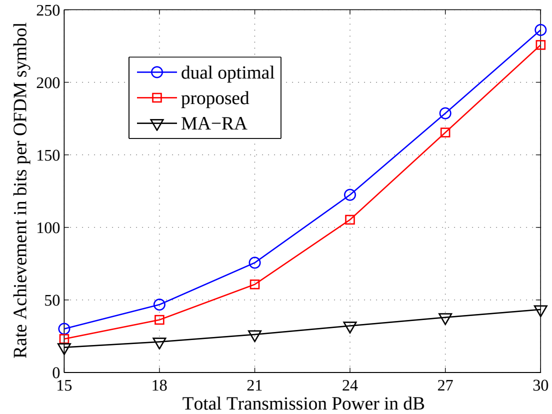

Fig. 2. Rate achievement vs. total transmission power.

C. Successive subcarrier adjustment

To make the performance loss small, a good starting point is offered by the above two algorithms for utilizing the efficient approaches from Section III in Algorithm 3, which outputs the subcarrier assignments. For each subcarrier m, three cases exist. Either, the subcarrier is used by an MA user, by the RA users, or not used by any user. A power increment is induced by excluding m from a subcarrier assignment, denoted by $\Delta P^{(\mathrm{r})}(m)$, see Section III. If no MA user uses subcarrier m, $\Delta P^{(\mathrm{r})}(m)$ is equal to 0. If the MA user using subcarrier m only has one subcarrier, then $\Delta P^{(\mathrm{r})}(m)$ must be set to ∞.

Set $\mathcal{U}$ contains the MA users, who may potentially use subcarrier m but do not use it presently. The power decrement, by adding m to the subcarrier assignments of those users, is referred to as $\Delta P_k^{(\mathrm{a})}(m)$. The user with the smallest power decrement in $\mathcal{U}$ is $\hat{k}$. The sum of the power increment $\Delta P^{(\mathrm{r})}(m)$ and the power decrement $\Delta P_k^{(\mathrm{a})}(m)$ is the power variation while adjusting subcarrier m among the MA users, denoted by $\Delta P_k^{(\mathrm{MA})}(m)$. When the transmission power for the MA users is larger than the total transmission power, we only adjust subcarrier m among the MA users. If the power variation $\Delta P_{\hat{k}}^{(\mathrm{MA})}(m)$ is negative, the subcarrier m is moved from subcarrier assignment $\mathcal{S}_k, k \in \mathcal{B}$, to the subcarrier assignment of MA user $\hat{k}$.

When the MA users need power less than the total transmission power, we adjust subcarrier m among the MA and RA users. If subcarrier m is not used by the RA users, two cases may occur. Either subcarrier m is assigned to MA user $\hat{k}$. The power decrement $\Delta P_{\hat{k}}^{(\mathrm{MA})}(m)$ results in the rate increment for the RA users, indicated by $\Delta R^{(\mathrm{p})}(-\Delta P_{\hat{k}}^{(\mathrm{MA})}(m), \beta)$. Or subcarrier m is assigned to the RA users. This can be viewed as two steps: first, the rate increment $\Delta R^{(\mathrm{a})}(m)$ by adding subcarrier m to $\mathcal{S}_{K+1}$, and then, the rate decrement $\Delta R^{(\mathrm{p})}(-\Delta P^{(\mathrm{r})}(m), \beta^{(\mathrm{a})}(m))$ by reducing the transmission power for the RA users by $\Delta P^{(\mathrm{r})}(m)$. This subcarrier adjustment is actually performed for the case resulting in a larger positive rate increment for the RA users, where $(x)^+ = \max(x, 0)$. When subcarrier m is used by the RA users, m is moved from $\mathcal{S}_{K+1}$ of the RA users to $\mathcal{S}_{\hat{k}}$ of MA user $\hat{k}$, if the sum of the rate decrement by excluding m from $\mathcal{S}_{K+1}$ and the rate increment by increasing power for the RA users by $-\Delta P_{\hat{k}}^{(\mathrm{a})}(m)$ is positive.

Note that when $s_{K+1} = 0$ holds, the rate increment after increasing power for the RA users is still zero. This is not included by Algorithm 3 due to the space limit. At last, given $\mathcal{S}_k$ by Algorithm 3, the achieved rate for the RA users is determined by SUWF$^{(\mathrm{RA})}$ like Algorithm 2. The complexity of Algorithm 3 is dominated by the calculation for the power variation and the rate variation. The rest is just to exchange subcarrier assignments and update water levels accordingly. At most K power variations or rate variations must be calculated for each subcarrier. The complexity of the third step is $\mathcal{O}(KN)$.

V. Simulation Results

In this section, the rate achievement by our method is compared to the dual optimum [8] to problem (1). For comparison, we use a reference method, denoted by MA-RA, by inheriting the idea from [11], where the power and rate are dual optimally allocated to the MA users by the method given in [3] and then the rest resource is assigned to the RA users. The complexities of these three methods are shown in Table I. The rate achievement against the number of subcarriers, the number of users and the increasing total transmission power are recorded. The frequency selective channel is modeled as consisting of 8 taps with an exponentially decaying profile. The taps are independently complex Gaussian distributed with zero mean and variance 1. The noise power is set to -10 dB. We make the number of MA users K and the number of RA users L equal varying from one to five ($K = L$). The OFDM system consists of 64 to 128 subcarriers and provides transmission with BER$^{(\mathrm{MA})} = 2.55 \times 10^{-3}$ to the MA users and transmission with BER$^{(\mathrm{RA})} = 2.63 \times 10^{-4}$ to the RA

TABLE I
Complexity Comparison

Proposed method	Dual method	MA-RA
$\mathcal{O}(KN)$	$\mathcal{O}((K+1)^3 N)$	$\mathcal{O}(K^3 N + KN)$

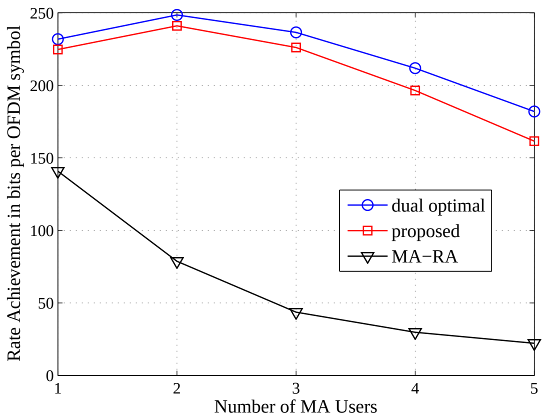

Fig. 3. Rate achievement vs. number of MA users.

users. One half of the MA users requires 64 bits per OFDM symbol and the other demands 16 bits per OFDM symbol. The maximal total transmission power at the base station is limited to 30 dB. A number of more than 5000 solvable channel samples are generated for each simulation.

When there are 3 MA users and 3 RA users, Fig. 1 shows the rate achievement for the RA users given different numbers of subcarriers with $P^{(\text{tot})} = 30\,\text{dB}$, and Fig. 2 draws the rate achievement for the RA users over different limits on the total transmission power with $N = 64$ subcarriers. Fig. 1 demonstrates that the performance loss of our method is almost constant compared to the dual method. In Fig. 2, the gap between the outputs by the dual method and our method becomes smaller at large numbers of subcarriers, while MA-RA gives much worse solution.

Fig. 3 gives the rate achievement against the number of users K with 64 subcarriers and $P^{(\text{tot})} = 30\,\text{dB}$, where $K = L$ holds. The output of MA-RA is dramatically decreasing in K. As K increases, the rate demand of the MA users increases, while the diversity of channels among users is enlarged. The former cause dominates the performance of OFDM systems with a larger number of users. The latter one plays a main role on the performance, while the number of users is smaller. Hence, the outputs of the dual method and our method are increasing in the smaller K and decreasing in the larger K. It is shown by the above three figures that the achieved rate by our method is close to the one by the dual method. It is always much larger than the output rate by MA-RA.

VI. Conclusion

In this work, we have formulated the resource allocation problem for heterogeneous transmission in multiuser OFDM downlink. Resource allocation methods solving this problem have to be computationally efficient to adapt to the fast time-varying channel. First, we have investigated the efficient updating approaches while changing the subcarrier assignments for the MA and RA users or the transmission power for the RA users. Then, to utilize these efficient approaches, we have heuristically designed a computationally efficient method to solve the resource allocation problem formulated earlier. Its complexity is linearly increasing in the number of subcarriers and the number of users. Simulations have demonstrated that our method has slight performance loss compared to the dual method and achieves a better balance between the performance and the complexity compared to the previous work.

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