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Exercise 11

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Problem 1. *(Support Vector Machine with Only One Member per Class)* Let the dataset consist of only two points, $(\mathbf{x}_1, y_1 = +1)$ and $(\mathbf{x}_2, y_2 = -1)$. Find the SVM classifier and its parameters.

Problem 2. *(Support Vector Machine Margin)* Let the dataset consist of points, $(\mathbf{x}_i, y_i = +1)$, $i = 1, 2$ and $(\mathbf{x}_3, y_3 = -1)$. Suppose that these points are linearly separable.

a) Show that if these points form an obtuse triangle, the maximum margin of the SVM classifier is obtained by the minimum of $\|\mathbf{x}_1 - \mathbf{x}_3\|$ and $\|\mathbf{x}_2 - \mathbf{x}_3\|$.

b) Discuss the case where the points form an acute triangle and argue why the margin cannot be as the above case.

Problem 3. *SVM equivalent formulations:* Consider the SVM problem formulated as follows for a linearly seprable dataset:

$$\arg \min_{\mathbf{a}\in\mathbb{R}^p, b\in\mathbb{R}} \frac{1}{2}\|\mathbf{a}\|^2 \quad \text{s.t.} \quad y_i(\mathbf{a}^T\mathbf{x}_i + b) \geq 1, \ i = 1, \ldots, n$$

Show that this problem is equivalent to the following problems.

a)

$$\arg \max_{\mathbf{a}\in\mathbb{R}^p, b\in\mathbb{R}, \|\mathbf{a}\|=1} \min_{i\in\{1,\ldots,n\}} y_i(\mathbf{a}^T\mathbf{x}_i + b)$$

b)

$$\arg \max_{\mathbf{a}\in\mathbb{R}^p, b\in\mathbb{R}, \|\mathbf{a}\|=1} \min_{i\in\{1,\ldots,n\}} |\mathbf{a}^T\mathbf{x}_i + b| \quad \text{s.t.} \quad y_i(\mathbf{a}^T\mathbf{x}_i + b) > 0, \ i = 1, \ldots, n$$