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Problem 1. (Basic Entropy)

Consider two random experiments with three outcomes respectively. The corresponding probability mass functions (e.m.f) are given by

a) (0.9,0.05,0.05)

b) (0.4,0.3,0.3)

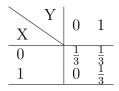
Find the entropy for a) and b)?

Problem 2. (Minimum entropy)

Let $X \in \{1, ..., n\}$ be a random variable and $p_i = P(X = i)$. What is the minimum value of $H(X) = H(p_1, ..., p_n) = H(\mathbf{p})$ as \mathbf{p} ranges over the set of *n*-dimensional probability vectors? Find all \mathbf{p} 's that achieve this minimum.

Problem 3. (Joint entropy)

Let the joint distribution of two random variables X and Y, denoted by p(x, y), be given in the following table:



Find

- **a)** H(X), H(Y),
- **b)** H(X|Y), H(Y|X),
- c) H(X,Y),
- **d)** H(Y) H(Y|X).

Problem 4. (Entropy of functions of a random variable)

Let X be a discrete random variable. Show that the entropy of a deterministic function of X is less than or equal to the entropy of X by justifying the following steps:

$$H(X, g(X)) \stackrel{(a)}{=} H(X) + H(g(X)|X)$$
$$\stackrel{(b)}{=} H(X),$$
$$H(X, g(X)) \stackrel{(c)}{=} H(g(X)) + H(X|g(X))$$
$$\stackrel{(d)}{\geq} H(g(X)).$$

Thus, $H(X) \ge H(g(X))$.

Problem 5. (Entropy of functions)

Let X be a random variable taking on a finite number of values. What is the (general) inequality relationship of H(X) and H(Y) if

a) Y = 2^X?
b) Y = cosX?

Problem 6. (Coin flips)

A fair coin is flipped until the first head occurs. Let X denote the number of flips required. Find the entropy H(X) in bits.

Hint: For 0 < r < 1 we have

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}, \quad \sum_{n=0}^{\infty} nr^n = \frac{r}{(1-r)^2}.$$