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Tutorial 4

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Problem 1. *(Chain Rule)*

Let $X_1, \ldots, X_n$ and Y be discrete random variables. Proof that

a)

$$H(X_1, \ldots, X_n) = \sum_{i=1}^{n} H(X_i|X_{i-1}, \ldots, X_1)\,,$$

b)

$$I(X_1, \ldots, X_n; Y) = \sum_{i=1}^{n} I(X_i; Y|X_{i-1}, \ldots, X_1)\,.$$

Let $Z = \{(X_n, Y_n)\}_{n\in\mathbb{N}}$ be an i.i.d. sequence of pairs of discrete random variables.

c) Show that

$$I(X_1, \ldots, X_n; Y_1, \ldots, Y_n) = \sum_{i=1}^{n} I(X_i; Y_i)\,.$$

Problem 2. *(Stationary Processes)*

Let $\ldots, X_{-1}, X_0, X_1, \ldots$ be a stationary stochastic process. Which of the following are true? Prove or provide a counterexample.

a) $H(X_n|X_0) = H(X_{-n}|X_0)$.

b) $H(X_n|X_0) \geq H(X_{n-1}|X_0)$.

c) $H(X_n|X_1, X_2, \ldots, X_{n-1}, X_{n+1})$ is non-increasing in n.

d) $H(X_n|X_1, \ldots, , X_{n-1}, X_{n+1}, \ldots, X_{2n})$ is non-increasing in n.

Problem 3. *(The past has little to say about the future)*

For a stationary stochastic process $X_1, X_2, \ldots$, show that

$$\lim_{n\to\infty} \frac{1}{2n} I(X_1, \ldots, X_n; X_{n+1}, \ldots, X_{2n}) = 0\,.$$

Problem 4. *(Entropy Rate)*

Let $X = \{X_n\}_{n\in\mathbb{N}}$ be a stationary sequence of discrete random variables with entropy rate $H_\infty(X)$.

a) Show that $H_\infty(X) \leq H(X_1)$.

b) What are the conditions for equality?