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Tutorial 6

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Problem 1. *Differential entropy*

Evaluate the differential entropy $h(X)$ for the following:

a) Guassian distributions with density, $f(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp(\frac{-(x-\mu)^2}{2\sigma^2})$.

b) The exponential density, $f(x) = \lambda\exp(-\lambda x), x \geq 0$.

c) The Laplace density, $f(x) = \frac{1}{2}\lambda\exp(-\lambda|x|)$.

Problem 2. *Channel with uniform distributed noise*

Consider a additive channel whose input alphabet $\mathcal{X} = \{0, \pm 1, \pm 2\}$ and whose output $Y = X + Z$, where Z is distributed uniformly over the interval $[-1, 1]$. Thus, the input of the channel is a discrete random variable, where as the output is continuous. Calculate the capacity $C = \max_{p(x)} I(X, Y)$ of this channel.

Problem 3. *Quantized random variables*

Roughly how many bits are required on the average to describe to three-digit accuracy the decay time (in years) of a radium atom if the half-life of the radium is 80 years? Note: The half-life is the median of the distribution.

Problem 4. *Shape of the typical set*

Let X_i be i.i.d $\sim f(x)$ where

$$f(x) = ce^{-x^4}. \tag{1}$$

Let $h = -\int f \ln f$. Describe the shape/form or the typical set $A_\epsilon^n = \{(x_1, x_2, ..., x_n) \in \mathcal{R}^n : f(x_1, x_2, ..., x_n) \in (2^{-n(h+\epsilon)}, 2^{-n(h-\epsilon)})\}$.