RWTH AACHEN UNIVERSITY

Lehrstuhl für Theoretische Informationstechnik

Review Exercise Cryptography I

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Problem 1.

a) What does a Friedman Test decide?

b) Compute the expectation of the index of coincidence over the alphabet $\mathcal{A} = \{A, B, C\}$ if the sequence of characters in the ciphertext is independent and identically distributed and each character is uniformly distributed.

Consider a Vigenère cipher with message space $\mathcal{M} = \mathcal{A}^n$, $n \in \mathbb{N}$, and key space $\mathcal{K} = \mathcal{A}^w$, $w \in \mathbb{N}$, with $w \mid n$. The messages and keys are both uniformly distributed.

c) Why has the cryptosystem perfect secrecy for $w = n$? Why has it no perfect secrecy for $w < n$?

d) Estimate the index of coincidence $I_{\mathbf{c}}$ over the alphabet $\mathcal{A}$ for the following ciphertext $\mathbf{c}$ of length $n = 24$:

$$\mathbf{c} = \text{CAAABBCACBCABACAABCCCACA}.$$

e) Assume the characters of the plaintext occur with the following frequencies: $\#A = 16$, $\#B = 5$ and $\#C = 3$. Determine the key $\mathbf{k}$ of the Vigenère cipher and decrypt the ciphertext $\mathbf{c}$ for $w = 4$ by identifying the frequencies from the ciphertext.

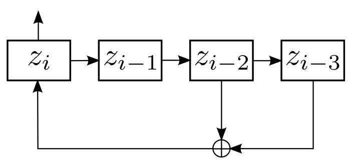
f) State four more classical cryptosystems from the lecture.

Problem 2.

A key stream $\mathbf{z} = (z_i)_{i\in\mathbb{N}}$ is generated from a key $\mathbf{k} = (k_1 \dots k_q)$ by the following recursion:

$$\begin{aligned} z_i &= k_i, && 1 \le i \le q, \\ z_i &= \textstyle\sum_{j=1}^{q} p_j z_{i-j}, && q < i. \end{aligned}$$

All computations are in the finite field $\mathbb{F}_2$. Consider the following *linear feedback shift register* (LFSR) for stream ciphers to generate a key stream:



a) Derive the feedback polynomial $p(x) = 1 + \sum_{i=1}^{q} p_i\, x^i$ of the LFSR given in the figure above. Show that the polynomial $p(x)$ is primitive[1] in $\mathbb{F}_2$ to ensure that the LFSR has the maximal period.

The plaintext symbols m_i are encrypted into the ciphertext symbols c_i as follows:

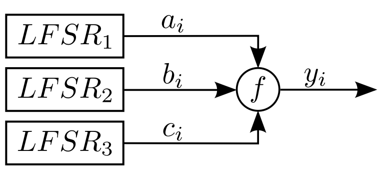
$$c_i = z_i \oplus m_i.$$

b) A key $\mathbf{k}$ is called *weak*, if $\mathbf{c} = \mathbf{m}$ for all $m \in \mathcal{M}$ holds. Find a weak key for the given LFSR.

c) Encrypt $\mathbf{m} = (m_1 \ldots m_{10}) = (0100101101)$ with the key $\mathbf{k} = (k_1 k_2 k_3) = (100)$.

d) What is the length of the (maximal) period of $\mathbf{z}$? How many zeros and ones occur within one period?

Now consider the following key stream generator with three independent LFSRs of period lengths l_1, l_2, l_3:



Answer the following questions without giving an explicit proof:

e) What is the period length of the sequence of triples $((a_i, b_i, c_i))_{i \in \mathbb{N}}$ for the key stream generator? What are the minimal and maximal period lengths? How should l_1, l_2, l_3 be chosen to maximize the period length?

The function f is defined by:

$$y_i = f(a_i, b_i, c_i) = (a_i \wedge b_i) \oplus (\overline{a_i} \wedge c_i).$$

f) Compute the probabilites $\Pr(b_i \mid y_i)$ assuming that (a_i, b_i, c_i) is uniformly distributed over $\mathbb{F}_2^3$? Which estimate for b_i ensures a success probability of $\frac{3}{4}$, if y_i is given?

Problem 3.

In the following an RSA cryptosystem with public key (n, e) is considered.

a) Does a valid RSA public key exist with $e = 2$? Substantiate your answer.

The message m is encrypted by means of the public key $(n, e) = (4819, 3343)$ resulting in the cryptogram $c = 1219$.

b) Factorize n and determine the private key.

c) What is the original message m?

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[1] A polynomial $p(x)$ of degree q is called *primitive* if and only if the smallest $n \in \mathbb{N}$ for which $p(x)$ divides the polynomial $x^n + 1$ is $n = 2^q - 1$.

In an RSA cryptosystem the public key $(n', e') = (391, 7)$ and the corresponding private key $d' = 151$ are known.

d) Compute the prime factors p and q of n' by means of the following steps:

   i) Determine a multiple x of $\varphi(n')$, i.e., an $x \in \mathbb{Z}$ with $x = k \cdot \varphi(n')$ for a $k \in \mathbb{N}$.

   ii) Compute the prime factorization of x.

   iii) Use the prime factorization to determine k, p, and q.