

c) Verification of step (i) of ElGamal signatures requires checking of $1 \leq r \leq p-1$

If this check is omitted, then Oscar can sign messages of his choice provided he has one valid signature and $h(m)^{-1} \pmod{p-1}$ should exist.

Suppose (r, s) is a signature for message m .

O selects a message m' of his choice and computes

$$h(m') \text{ and } u = h(m') (h(m))^{-1} \pmod{p-1}$$

$$r' = su \pmod{p-1}$$

$$r' \text{ such that } r' \equiv ru \pmod{p-1} \text{ and } r' \equiv r \pmod{p}$$

Solve this by CRT, because $p, p-1$ are relatively prime

The pair (r', s) is a signature for m' which would be accepted if $1 \leq r' \leq p-1$ is ignored.

11.2 The Digital Signature Algorithm (DSA)

- Proposal by the NIST in Aug '91
- Standardized as FIPS 186, named DSS (Digital Signature Standard)
- Developed by the NSA (not publicly)
- DSA is a variant of the ElGamal signature scheme
- Needs a hash function $h: \{0,1\}^* \rightarrow \mathbb{Z}_q$ as a building block
The standard prescribes SHA-1

System parameters

Each user generates a public and private key as follows:

1. Choose a prime q with $2^{159} < q < 2^{160}$ (160 bits)
2. Choose t , $0 \leq t \leq 8$, further a prime p such that
 $2^{511+64t} < p < 2^{512+64t}$ and $q | p-1$ (512... 1024 bits)

(Recommended by NIST from Oct 2001: $t=8$, 1024 bits)

3. (i) Select $g \in \mathbb{Z}_p^*$, compute $a = g^{(p-1)/q} \pmod p$

(ii) If $a=1$, repeat step (i)

(a is a generator of a cyclic subgroup of order q in \mathbb{Z}_p^*)

4. Choose some random $x \in \{1, \dots, p-1\}$ (Could be $q-1$ as well)

5. Compute $\gamma = a^x \pmod p$

6. Public key: (p, q, a, γ) , Private key x

Signing a message $m \in \{0,1\}^*$

1. Choose a random $k \in \{1, \dots, q-1\}$

2. $r = (a^k \pmod p) \pmod q$

3. Compute $k^{-1} \pmod q$

4. $s = k^{-1}(h(m) + x \cdot r) \pmod q$

5. Signature (r, s) (320 bits in total)

Verification of signature (r, s) on message m :

1. Check if $0 < r < q$ and $0 < s < q$, otherwise decline
2. $w = s^{-1} \pmod{q}$
3. $u_1 = (w h(m)) \pmod{q}$, $u_2 = (r \cdot w) \pmod{q}$
4. $v = (a^{u_1} \cdot \gamma^{u_2} \pmod{p}) \pmod{q}$
5. Accept the signature if $v = r$

Proof that the verification is correct:

For a valid signature (r, s) it holds that

$$h(m) \equiv k \cdot s - x \cdot r \pmod{q}$$

Hence, $a^{u_1} \cdot \gamma^{u_2} \equiv a^{u_1 + x u_2} \pmod{p}$

$$u_1 + x u_2 \equiv w h(m) + x r w \equiv w k \cdot s - w x r + x r w \equiv k \pmod{q}$$

$$v \equiv \left(\underbrace{a^{kq+k}}_{\substack{a \text{ has order } q, \text{ i.e., } a^q \equiv 1 \pmod{p}}} \pmod{p} \right) \pmod{q} = \left(a^k \pmod{p} \right) \pmod{q} = r \quad \checkmark$$

Security

- Security relies on ~~the~~ two DL problems
 - a) in \mathbb{Z}_p^*
 - b) in $\langle a \rangle \leq \mathbb{Z}_p^*$ ($\langle a \rangle$ denotes the subgroup gen. by a)
- Security principles of the ElGamal scheme carry over:
 - always choose ~~as~~ a new k
 - use of hash functions is mandatory
 - always verify 1. in the verification procedure. Otherwise signatures for arbitrary messages can be generated provided one valid signature is known.

Remarks

- a) Modular exponentiation is in the range of q (160 bits)
(rather than 1024 El Gamal)
- b) $k, k^{-1}, r, x+r$ may be generated, computed and stored in advance
- c) Verification needs 2 instead of 3 modular exponentiations
- d) Signature by DSA is short, 320 bits, instead of 2048 bits for El Gamal.
- e) In the verification step, also check, if $r \neq 0, s \neq 0$, otherwise the signature is rejected. But this happens with a very small probability.

12. Identification and Entity Authentication

This chapter considers techniques to allow the "verifier" to establish the identity of the "claimant", thereby preventing impersonation.

Requirements on authentication protocols:

1. A is able to uniquely identify herself to B
2. B cannot reuse an identification exchange with A so as to impersonate A to a third party C. (Transferability)
3. It is practically infeasible that a third party C can cause B to wrongly accept the identity of A. (Impersonation)
4. Even if C observes the identification process between A and B very often he cannot impersonate A.

Three main categories of identification:

1. Something is known: password, PIN, private key
2. Something possessed: key, magnetic-stripped cards, chipcards, PIN or password generators, ...
3. Something inherent: human physical characteristics, face recognition, fingerprint, retinal patterns, hand-written signatures

12.1 Passwords

Fixed password schemes

Rather than storing a cleartext user password pwd in a file, a hash value $h(pwd)$ of each user password is stored. Verification is done by comparing the hash value of the entered password with the stored one for a given user.

Main attacks are :

- replay of fixed passwords
- exhaustive password search
- password-guessing and dictionary attacks

Defense strategies are

- Choose a random password, or nearly random, use of special characters (increasing entropy)
- Slowing down the password mapping
- Salting passwords

Extend the password by some random string, the salt, before hashing. Both the hashed password and the salt are stored

$h(\text{password}, \text{salt}), \text{salt}$

This does not complicate exhaustive search, but, simultaneous dictionary attacks against a large set of passwords

One-time passwords

Protects against eavesdropping and replay of passwords or "phishing".

Canter's protocol

Objective: A identifies herself B

Use a ~~secret~~ one way function H

Notation: $H^k(w) = \underbrace{H(H(\dots H(w)))}_{k\text{-times}}$

Initial parameters: t : max number of identifications ($t=100, 1000$)

A chooses an initial password w

A transfers $w_0 = H^t(w)$ to B

B initializes his counter for A to $i_A = 1$

Protocol actions for session i :

A computes $w_i = H^{t-i}(w)$, transfers to B: (A, i, w_i)

B checks that $i = i_A$ and $w_{i-1} = H(w_i)$. If both checks succeed B accepts and sets $i_A \leftarrow i_A + 1$ and stores w_i