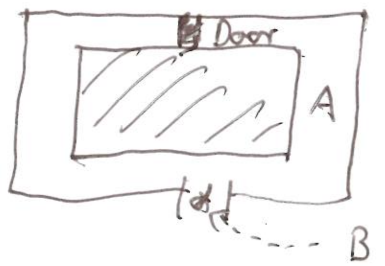
12.4 Zero Knowledge Identification protocols

Demonstrative Example



A proves to B that she can unlock the door (without giving away any information how she does it)

- A enters the tunnel and goes either to the left or right
- B waits, stands at ✱, and calls randomly „left" or „right"
- A appears from the left or the right, as requested
- If A comes from the right direction for each of n repetitions there is only a probability of 2^{-n} that she does not know how to open the door.
- O/E sets up a video camera at ✱, will gain no information to convince others tat O/E can go through the door

General structure of zero-knowledge protocols

1. A → B : witness : A selects a random element, from this computes a public witness : Purpose
   - variation from other protocol runs
   - defines a set of questions, answerable only by A
2. A ← B : challenge : B selects a question
3. A → B : response : A answers the question, B checks correctness

Example

Let $n = pq$, p, q prime

A selects random s, computes $y = s^2 \pmod{n}$ with $\gcd(y, n) = 1$

A claims to know a square root of y without revealing s

Protocol

1. A chooses randomly r_1, r_2 with

$$r_1 \cdot r_2 \equiv s \pmod{n}$$

by: choose r_1 at random with $\gcd(r_1, n) = 1$

let $r_2 = s \cdot r_1^{-1} \pmod{n}$

Compute $x_1 = r_1^2 \pmod{n}$ $x_2 = r_2^2 \pmod{n}$

$A \to B : (x_1, x_2)$ (witness)

2. B checks if $x_1 \cdot x_2 \equiv y \pmod{n}$

B chooses randomly either x_1 or x_2

B asks A to supply a square root of it. (challenge)

3. A sends the square root, e.g. r_1

B checks if it is a square root by $r_1^2 \equiv x_1 \pmod{n}$

Iterate this protocol t times, because O/E have a 50% chance of giving ~~the~~ a correct answer.

Ex.: Discuss the protocol

12.4.1 Feige – Fiat – Shamir Identification Protocol (1988)

Relies on the hardness of computing square roots mod n, n composite

Objective: A proves her identity to B

System parameters

(i) A, trusted authority (TA), publishes $n = p \cdot q$, $p, q \equiv 3 \pmod 4$

(ii) Each entity A selects random number $s_1, \ldots, s_k \in \{1, \ldots, n-1\}$ $\gcd(s_i, n) = 1$, computes $v_i = (s_i^2)^{-1} \pmod n$ publishes $v_1, \ldots, v_k$

Protocol actions

1. A chooses a random integer r, computes $x = r^2 \bmod n$

   $A \to B : x$ (witness)

2. B chooses random bits $b_1, \ldots, b_k \in \{0, 1\}$

   $A \leftarrow B : (b_1, \ldots, b_k)$ (challenge)

3. A computes $y = r \prod_{j=1}^{k} s_j^{b_j} \bmod n$

   $A \to B : y$ (response)

4. B checks that $y^2 \prod_{j=1}^{k} v_j^{b_j} \equiv x \pmod n$

Security aspects

Oscar wants to impersonate A

Suppose O guess $(b_1, \ldots, b_k)$ before he sends x.

O chooses a random integer $a \in \{1, \ldots, n-1\}$, computes

$x = a^2 \prod_{j=1}^{k} v_j^{b_j} \bmod n$

O sends in step 3 $O \to B : a$

B checks in 4 that $a^2 \prod_{j=1}^{k} v_j^{b_j} \equiv x \pmod n$ accepts A's identity

However the probability to guess $(b_1, \ldots, b_k)$ correctly in t trials is $\frac{1}{2^{tk}}$

An identification scheme based on the FFS identification protocol

I_A : identification string for A, containing, e.g., ~~name~~, birthday, etc

Notation: $I_A \| j$ concatenation, h some hash function

TA computes $h(I_A \| j)$ for some j until it receives integers
$v_1 = h(I_A \| j_1), \ldots, v_k = h(I_A \| j_k)$ with square roots
$s_1, \ldots, s_k \pmod n$ computed by knowing p, q.

$I_A, n, j_1, \ldots, j_k$

$s_1, \ldots, s_k$ are given to A (and kept secret)

Identification to an ATM, e.g.,

- ATM reads I_A from A's card
- download $n, j_1, \ldots, j_k$ from a data base
- calculate $v_1 = h(I_A \| j_1), \ldots, v_k = h(I_A \| j_k)$
- perform the preceeding protocol t times

12.4.2 Schnorr Identification Protocol

Obj.: A proves her identity to B
Relies on hardness of computing discrete logs.

System parameters

1. A trusted authority chooses:
   - p prime, q prime, $q \mid p-1$ $(p \simeq 2^{1024}, q \geq 2^{160})$
   - $\beta \in \mathbb{Z}_p^*$ of order q
   - TA publishes and signs p, q, β
   - Security parameter t with $2^t < q$
2. Each user A
   - chooses a private key a $0 \leq a \leq q-1$
   - computes $v = \beta^{-a} \mod p$
   - publishes v (TA signs (A, v) after securing the id. of A)

Protocol actions

1. A chooses a random number $r \in \{1, \dots q-1\}$

   $A \to B : x = \beta^r \mod p$ (witness)
2. B chooses a random number $e \in \{1 \dots, 2^t\}$

   $A \leftarrow B : e$ (challenge)
3. A checks $1 \leq e \leq 2^t$

   $A \to B : y = (a \cdot e + r) \mod q$ (response)
4. B computes $z = \beta^y v^e \mod p$

   verifies $z = x$

Remarks

a) Protocol is correct since

$$\beta^y v^e \equiv \beta^{(a\cdot e + r) \bmod q}\, \beta^{-a\cdot e} \overset{l \in \mathbb{Z}}{\equiv} \beta^{a\cdot e + r + l\cdot q} \cdot \beta^{-ae}$$

$$\equiv \beta^r \equiv x \pmod p \qquad \text{// as } \beta \text{ has order } q \text{ in } \mathbb{Z}_p^*$$

b) Suppose O/E guesses e prior to sending x

O chooses some y, computes $x = \beta^y \cdot v^e \bmod p$, sends

in 1 $O \to B : x$

in 3 $O \to B : y$

Then $z \equiv \beta^y v^e \equiv x \pmod p$, B accepts in 4 O's identity

c) The protocol is particularly suited for smart cards

computational effort

in 1: fast exponentiations (expensive, but may be computed in advance)

in 3: one modular multipl. and addition (cheap)