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Exercise 12

Friday, July 15, 2016

Problem 1. (*How not to use the ElGamal cryptoystem*) Alice and Bob are using the ElGamal cryptosystem. The public key of Alice is $(p, a, y) = (3571, 2, 2905)$. Bob encrypts the messages m_1 and m_2 as

$$\mathbf{C}_1 = (1537, 2192) \text{ and } \mathbf{C}_2 = (1537, 1393).$$

a) Show that the public key is valid.

b) What did Bob do wrong?

c) The first message is given as $m_1 = 567$. Determine the message m_2.

Problem 2. *(Euler's criterion)* Prove Euler's criterion (Proposition 9.2): Let $p > 2$ be prime, then

$$c \in \mathbb{Z}_p^* \text{ is a quadratic residue modulo } p \Leftrightarrow c^{\frac{p-1}{2}} \equiv 1 \mod p\,.$$

Problem 3. *(properties of quadratic residues)* Let p be prime, g a primitive element modulo p and $a, b \in \mathbb{Z}_p^*$. Show the following:

a) a is a quadratic residue modulo p if and only if there exists an even $i \in \mathbb{N}_0$ with $a \equiv g^i \mod p$.

b) If p is odd, then exactly one half of the elements $x \in \mathbb{Z}_p^*$ are quadratic residues modulo p.

c) The product $a \cdot b$ is a quadratic residue modulo p if and only if a and b are both either quadratic residues or quadratic non-residues modulo p.

Problem 4. *(Rabin cryptosystem)* Alice and Bob are using the Rabin Cryptosystem. Bob uses the public key $n = 4757 = 67 \cdot 71$. All integers in the set $\{1, \ldots, n-1\}$ are represented as a bit sequence of 13 bits. In order to be able to identify the correct message, Alice and Bob agreed to only send messages with the last 2 bits set to 1. Alice sends the cryptogram $c = 1935$. Decipher this cryptogram.