

### 5.2.4. Design Considerations and Security

- After 2 rounds full diffusion is achieved, i.e., if 1 byte is modified in the input all 16 bytes of the output are modified.

- S-box is constructed as  $x \mapsto x^{-1}$  in  $\mathbb{F}_{28}$ .

#### Advantages

- simple, algebraic, highly nonlinear
- resisting differential and linear cryptanalysis
- no suspicion about trapdoors  
(no mysteries about trapdoors)

- Shift Rows to resist 2 attacks: "truncated differentials" and "square attack"

- Mix Columns causes diffusion

- Key schedule to avoid advantage from partial knowledge.

- Presently no better attacks than exhaustive search against AES 128.

Attacks against AES 142 and AES 256 of complexity  $\sim 2^{119}$

see lecture notes.

### 5.3. Other block ciphers

- o IDEA - Internat. Data Encryption Alg.
  - designed by Lai & Massey, 1990, Ascona, Switzerland
  - IDEA was part of PGP
  - Block length 64 Bits, Key length 128 Bits  
(description, see Schneier, p. 319)
  - IDEA is secure, best known attack exhaustive search
  - Patented in EU (1991), USA (1993)  
Non-commercial applications are free.
- o RC5 (Ronald Rivest, 1994)
- o Blowfish (B. Schneier, 1993)
- o Serpent (Anderson, Biryukov, Knudsen, 1998)

## 5.4. Modes of Operation

Let  $BC_K$  be a block cipher operating on blocks of fixed length using key  $K$ .

5 modes of operations were standardized in Dec. 1980.

### 5.4.1. ECB (electronic codebook mode)

Direct use of  $BC_K$ . Plaintext blocks  $M_1, M_2, M_3, \dots$

Encryption:  $C_i = BC_K(M_i), i=1,2,\dots$

Decryption:  $M_i = BC_K^{-1}(C_i),$

### 5.4.2. CBC (cipher block chaining mode)

Given: Plaintext blocks  $M_1, M_2, \dots$   
 Key  $K$   
 Initial vector (IV)  $C_0$  (non-secret) } (\*)

Encryption:  $C_i = BC_K(C_{i-1} \oplus M_i), i=1,2,\dots$

Decryption:  $C_{i-1} \oplus M_i = BC_K^{-1}(C_i),$  hence

$M_i = BC_K^{-1}(C_i) \oplus C_{i-1}, i=1,2,\dots$

### 5.4.3. OFB (output feedback mode)

Given (\*),  $Z_0 = C_0$

Encryption:  $Z_i = BC_K(Z_{i-1}), C_i = M_i \oplus Z_i$

Decryption: " ,  $M_i = C_i \oplus Z_i, i=1,2,\dots$

(mimics the one-time pad)

5.4.4. CFB (cipher feedback mode)

Given (\*)

Encryption:  $Z_i = BC_K(C_{i-1})$ ,  $C_i = M_i \oplus Z_i$ ,  $i=1,2,\dots$

Decryption:  $M_i = C_i \oplus Z_i = C_i \oplus BC_K(C_{i-1})$ ,  $i=1,2,\dots$

5.4.5. CTR (counter mode)

Given (\*),  $Z_0 = C_0$  (interpreted as some integer)

Encryption:  $Z_i = Z_{i-1} + 1$ ,  $C_i = BC_K(Z_i) \oplus M_i$

Decryption: " ,  $M_i = BC_K(Z_i) \oplus C_i$

Applications:

Example: MAC - message authentication code

In CBC and CFB modes, changing any plaintext block affects all subsequent ciphertext blocks.

Appropriate for generating a MAC.

- Append  $C_n$  to the message  $(M_1, \dots, M_n)$
- The authorized receiver, knowing  $K$ , can verify  $C_n$ .  $\rightarrow$  integrity or authenticity of  $(M_1, \dots, M_n)$

Example: Storing passwords.

Direct storing of passwords is insecure. Hence:

- User types (name, password)
- System generates a key  $K = K(\text{name}, \text{password})$  and stores (name,  $BC_K(\text{password})$ )
- When logging in, system compares (name,  $BC_K(\text{password})$ ) with the stored value.

Knowledge of (name,  $BC_K(\text{password})$ ) is useless for an intruder.

## 6. Number-Theoretic Reference Problems

Consider  $\mathbb{Z}_n$ : ring of equivalence classes mod  $n$

$$s, t \in \mathbb{Z}_n : s \sim t \text{ or } s \equiv t \pmod{n} \Leftrightarrow n \mid (s-t)$$

( $\sim$  is an equivalence relation on  $\mathbb{Z}$ )

$(\mathbb{Z}_n, +, \cdot)$  forms a ring

Def. 6.1.  $\mathbb{Z}_n^* = \{a \in \mathbb{Z}_n \mid \gcd(a, n) = 1\}$

is called the multiplicative group of  $\mathbb{Z}_n$ .

$\varphi(n) = \#\mathbb{Z}_n^*$  is called Euler- $\varphi$ -function.

( $\#$ : order of  $\mathbb{Z}_n^*$ , no. of elements of  $\mathbb{Z}_n^*$ ,  
cardinality)

Remarks

•  $\varphi(p) = p-1$  if  $p$  is prime.

•  $\mathbb{Z}_n^*$  is a multiplicative group (Abelian).

It holds

$\gcd(a, n) = 1 \Leftrightarrow \exists$  inverse  $s$  of  $a$ , i.e.,  $a \cdot s \equiv s \cdot a \equiv 1 \pmod{n}$ .

• Notation  $\gcd(a, n) = (a, n)$ . If  $(a, n) = 1$ ,  
 $a$  and  $n$  are called relatively prime or coprime.

Theorem 6.2. (Euler, Fermat)

If  $a \in \mathbb{Z}_n^*$ , then  $a^{\varphi(n)} \equiv 1 \pmod{n}$

In particular (Fermat's little theorem)

If  $p$  prime,  $(a, p) = 1$  then  $a^{p-1} \equiv 1 \pmod{p}$