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Exercise 13

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Problem 1.

(computing square roots modulo p) The following scheme is used to compute square roots modulo a prime number p.

Algorithm 1 Computing square roots modulo a prime number p.

Input: An odd prime number p and a quadratic residue a modulo p

Output: Two square roots $(r, -r)$ of a modulo p

1) Choose a random $b \in \mathbb{Z}_p$ until $v = b^2 - 4a$ is a quadratic non-residue modulo p.
2) Let $f(x)$ denote the polynomial $x^2 - bx + a$ with coefficients in $\mathbb{Z}_p$.
3) Compute $r = x^{\frac{p+1}{2}} \mod f(x)$ (Use without proof: r is an integer)

return $(r, -r)$

a) Let $p = 11$ and $a = 5$. Compute the square roots of a using Algorithm 1 above. Instead of choosing b at random, begin with $b = 5$. If b is invalid, increment b by one.
Hint: To compute r in step 3), perform the polynomial division.

Consider the Rabin cryptosystem. The prime numbers are given by $p = 11$ and $q = 23$. It is known that the plaintext message m ends with 0100 in its binary representation.

b) Decrypt the ciphertext $c = 225$.

c) Naive Nelson announces that the plaintext message m ends with 1111 in its binary representation. Why is this agreement a bad choice for the given ciphertext c?

Problem 2. *(Rabin cryptosystem)* Alice and Bob are using the Rabin Cryptosystem. Bob uses the public key $n = 4757 = 67 \cdot 71$. All integers in the set $\{1, \ldots, n-1\}$ are represented as a bit sequence of 13 bits. In order to be able to identify the correct message, Alice and Bob agreed to only send messages with the last 2 bits set to 1. Alice sends the cryptogram $c = 1935$. Decipher this cryptogram.

Problem 3. *(coin flipping)* Consider the coin flipping protocol. Let $p > 2$ be prime.

a) Show that if $x \equiv -x \mod p$, then $x \equiv 0 \mod p$.

b) Suppose Alice cheats when flipping coins over the telephone by choosing $p = q$. Show that Bob almost always loses if he trusts Alice.

c) Alice chooses $n = p^2$ as the secret key, but Bob suspects that Alice has cheated. Can Bob discover her attempt to cheat? Can Bob use Alice' cheating as an advantage for himself?