Lehrstuhl für Theoretische Informationstechnik

Homework 1 in Optimization in Engineering

Prof. Dr. Rudolf Mathar, Simon Görtzen, Markus Rothe

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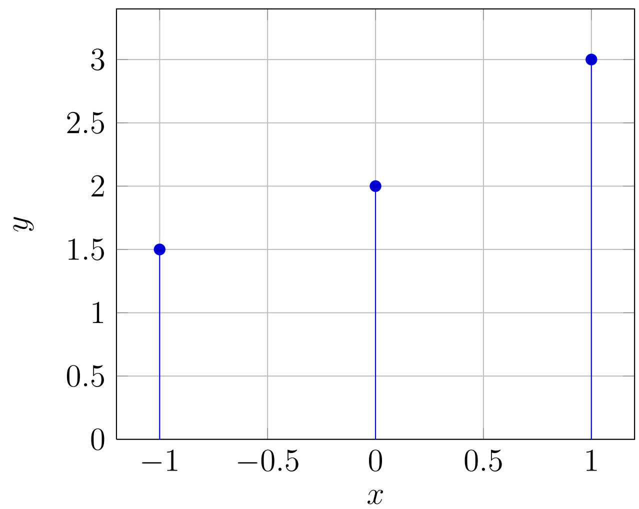
Exercise 1. (linear regression) In a series of measurements, the following three x-y-pairs have been obtained: $(-1, 1.5)$, $(0, 2.0)$, and $(1, 3.0)$.

Apart from measurement errors, y is assumed to be an affine function of x. The goal is to find a line $g(x) = m \cdot x + n$ which minimizes the sum of squared differences between $g(x)$ and y at all measurement points.

Formulate the above problem of linear regression as an optimization problem with optimization variable $\boldsymbol{v} = \binom{m}{n} \in \mathbb{R}^2$. Note the problem class and convert the problem to standard form if necessary. Compute the optimal solution $\boldsymbol{v}^*$.



Exercise 2. (convex functions) Show that the following functions are convex by verifying the definition, i.e., that

$$f(\alpha\boldsymbol{x} + (1-\alpha)\boldsymbol{y}) \leq \alpha f(\boldsymbol{x}) + (1-\alpha) f(\boldsymbol{y})$$

is satisfied for all $\boldsymbol{x}$, $\boldsymbol{y}$ in the domain of f and all $\alpha \in [0, 1]$.

a) $f(u) = u$, $u \in \mathbb{R}$

b) $f(u) = \frac{1}{u}$, $u \in \mathbb{R}_{>0}$

c) $f(u) = u^2$, $u \in \mathbb{R}$

d) $f(u) = |u|$, $u \in \mathbb{R}$