Homework 4 in Optimization in Engineering

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Exercise 1. (partial sum of convex sets) Show that if $\mathcal{S}_1$ and $\mathcal{S}_2$ are convex sets in $\mathbb{R}^{m+n}$, then so is their partial sum

$$\mathcal{S} = \{(x, y_1 + y_2) \,|\, x \in \mathbb{R}^m,\, y_1, y_2 \in \mathbb{R}^n,\, (x, y_1) \in \mathcal{S}_1,\, (x, y_2) \in \mathcal{S}_2\} \;.$$

Exercise 2. (invertible linear-fractional functions) Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be the linear-fractional function

$$f(x) = \frac{Ax + b}{c^T x + d}, \quad \operatorname{dom} f = \left\{x \middle| c^T x + d > 0\right\}.$$

Suppose the matrix

$$Q = \begin{bmatrix} A & b \\ c^T & d \end{bmatrix}$$

is nonsingular. Show that f is invertible and that f^{-1} is again a linear-fractional function. Give an explicit expression for f^{-1} in terms of Q.

Exercise 3. (convex norm cone) Let $x \in \mathbb{R}^n$, $t \in \mathbb{R}$. Suppose $||\cdot||$ is any norm in $\mathbb{R}^n$. The norm cone associated with this norm is the set

$$\mathcal{S} = \{(x, t) \,|\, ||x|| \leq t\} \;.$$

Show that the norm cone is convex.