Homework 9 in Optimization in Engineering

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Exercise 1. (dual problem bounds) For the following optimization problems with optimization variable $\boldsymbol{x} \in \mathbb{R}^2$, compute the dual problem and the maximum lower bound d^* for the optimal value p^*.

a)

$$\begin{array}{rl} \text{minimize} & 2x_1^2 + 8x_2^2 \\ \text{subject to} & 3x_1 + 6x_2 = 10 \end{array}$$

b)

$$\begin{array}{rl} \text{maximize} & 2x_1 x_2 \\ \text{subject to} & x_1^2 + x_2^2 = 1 \end{array}$$

Remark: Convert problem **b**) into a minimization problem first.

Exercise 2. (geometric interpretation of duality) For the optimization problems below, sketch the two sets

$$\mathcal{G} = \{(u,t) \,|\, \exists\, x \in \mathcal{D},\ f(x) = t,\ g(x) = u\} \text{ and}$$
$$\mathcal{A} = \{(u,t) \,|\, \exists\, x \in \mathcal{D},\ f(x) \le t,\ g(x) \le u\} \ .$$

Form the dual problem, solve both the primal and the dual problem, and answer the following three questions: Is the problem convex? Is *Slater's constraint qualification* satisfied? Does strong duality hold?

Remark: For problems **a**) to **e**), the domain is $\mathcal{D} = \mathbb{R}$.

a) minimize x subject to $x^2 \le 1$.

b) minimize x subject to $x^2 \le 0$.

c) minimize x subject to $|x| \le 0$.

d) minimize x subject to $\Gamma(x) \le 0$, with

$$\Gamma(x) = \begin{cases} -x+2, & 1 \le x \\ x, & -1 \le x \le 1 \\ -x-2, & x \le -1. \end{cases}$$

e) minimize x^3 subject to $-x+1 \le 0$.

f) minimize x^3 subject to $-x+1 \le 0$ with domain $\mathcal{D} = \mathbb{R}_{\ge 0}$.