Homework 10 in Optimization in Engineering

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Exercise 1. (sum rate maximization in OFDM) A single-user OFDM system provides n orthogonal subcarriers for data transmission. For subcarrier i, the data rate is computed from the signal-to-noise ratio $u_i = h_i/\sigma_i^2 \in \mathbb{R}_+$ as well as the transmit power p_i on this subcarrier as

$$r_i = \log(1 + p_i u_i)\,.$$

The goal of *sum rate maximization* is to find a transmit power vector $\boldsymbol{p} = (p_1, \ldots, p_n)$ such that the overall rate R, defined as the sum of the rates r_i of all subcarriers, is maximal. A global power budget P limits the combined power which can be spent over all subcarriers.

a) Formulate the sum rate maximization problem as a convex optimization problem in standard form.

b) State the KKT conditions for this problem. Derive an expression describing the relationship between a primal optimal $\boldsymbol{p}^*$ and a dual optimal $\boldsymbol{\lambda}^*$, respectively.

Exercise 2. (sum rate maximization in OFDMA) In a multi-user OFDM system, k users compete for the available n subcarriers. The parameters are the signal-to-noise ratios $u_{i,j}$ for subcarrier i and user j, as well as a weight vector $\boldsymbol{w} = (w_1, \ldots, w_k) \in \mathbb{R}_+^k$ with $\sum_{j=1}^k w_j = 1$.
The goal of *weighted sum rate maximization* is to find transmit powers $p_{i,j}$ for subcarrier i and user j such that the weighted sum rate $\sum_{j=1}^k w_j R_j$ is maximized. A global power budget P once again limits the combined transmit power, and each subcarrier can only be used by a single user.

a) Formulate the weighted sum rate maximization problem.

b) Assume that an allocating function $a\colon \{1, \ldots, n\} \to \{1, \ldots, k\}$ assigns each subcarrier i to a user $a(i)$. On the basis of this allocation, state the KKT conditions for the problem. Derive an expression describing the relationship between a primal optimal $\boldsymbol{p}^*$ and a dual optimal $\boldsymbol{\lambda}^*$, respectively.

c) Given a fixed allocation a, how does the solution to the multi-user problem differ from the solution to the single-user case?