Homework 11 in Optimization in Engineering

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Exercise 1. (exact line search) Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be the quadratic funtion

$$f(\boldsymbol{x}) = \frac{1}{2}\left(x_1^2 + \gamma x_2^2\right)$$

with $\gamma > 0$. Show that the minimization of f using a descent method with exact line search and start point $x^{(0)} = (\gamma, 1)$ leads in the k^{th} iteration to

$$x_1^{(k)} = \gamma \left(\frac{\gamma - 1}{\gamma + 1}\right)^k ,$$
$$x_2^{(k)} = \left(-\frac{\gamma - 1}{\gamma + 1}\right)^k .$$

Exercise 2. (convergence behaviour of Newton method) Newton's method with fixed step size $t = 1$ can diverge if the inital point is not close to x^*. In this problem we consider two examples.

a) $f(x) = \log\left(e^x + e^{-x}\right)$ has a unique minimizer $x^* = 0$. Run Newton's method with fixed step size $t = 1$, starting at $x^{(0)} = 1$ and at $x^{(0)} = 1.1$.

b) $f(x) = -\log x + x$ has a unique minimizer $x^* = 1$. Run Newton's method with fixed step size $t = 1$, starting at $x^{(0)} = 3$.

Plot f and f', and show the first few iterates.