Homework 4 in Optimization in Engineering

Prof. Dr. Anke Schmeink, Michael Reyer, Alper Tokel

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Exercise 1. (Second-order condition for convexity) Let $f : \mathcal{C} \to \mathbb{R}^n$ be a twice differentiable function on a convex set $\mathcal{C} \subset \mathbb{R}^n$. Prove the following statements.

(a) Let $n = 1$, then f is convex, iff $f''(x) \geq 0$, $\forall x \in \mathcal{C}$.

(b) f is convex, iff $\nabla^2 f(\boldsymbol{x}) \geq 0$, $\forall \boldsymbol{x} \in \mathcal{C}$.

Exercise 2. (Epigraph) Let $f \colon \mathcal{C} \to \mathbb{R}$ be a function defined on a convex, non-empty set $\mathcal{C} \subseteq \mathbb{R}^n$. Show that f is convex if and only if the epigraph of f

$$\operatorname{epi}(f) = \{(\boldsymbol{x}, y) \in \mathcal{C} \times \mathbb{R} \,|\, f(\boldsymbol{x}) \leq y\}$$

is a convex set.

Exercise 3. (Convex and concave functions) Decide which of the following functions are convex or concave and give reasons.

(a) $f(x) = |x|$, $x \in \mathbb{R}$

(b) $f(\boldsymbol{x}) = \|\boldsymbol{x}\|^p$, $\boldsymbol{x} \in \mathbb{R}^n$ and $p \geq 1$

(c) $f(x) = e^x - 1$, $x \in \mathbb{R}$

(d) $f(\boldsymbol{x}) = x_1 x_2$, $\boldsymbol{x} \in \mathbb{R}^2_{>0}$

(e) $f(\boldsymbol{x}) = \frac{1}{x_1 x_2}$, $\boldsymbol{x} \in \mathbb{R}^2_{>0}$