Homework 6 in Optimization in Engineering

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Exercise 1. (Channel capacity) In section 4.2 the calculation of a capacity achieving input distribution $\boldsymbol{p}$ for a discrete memoryless channel has been proven to be a convex optimization problem. The following channel matrices are given.

- $m_1 = d_1 = 2$, $\Pi_1 = \begin{bmatrix} 0.95 & 0.05 \\ 0.05 & 0.95 \end{bmatrix}$
- $m_2 = d_2 = 2$, $\Pi_2 = \begin{bmatrix} 0.80 & 0.20 \\ 0.05 & 0.95 \end{bmatrix}$
- $m_3 = d_3 = 4$, $\Pi_3 = \frac{1}{100} \begin{bmatrix} 90 & 4 & 4 & 2 \\ 2 & 76 & 20 & 2 \\ 2 & 20 & 76 & 2 \\ 2 & 4 & 4 & 90 \end{bmatrix}$

Tackle the following tasks by formulating and solving the optimization problems by means of the cvx-package in Matlab.

(a) What are the optimal input distributions?

(b) What are the channel capacities?

(c) What are the data rates in bits per symbol?

Hints:

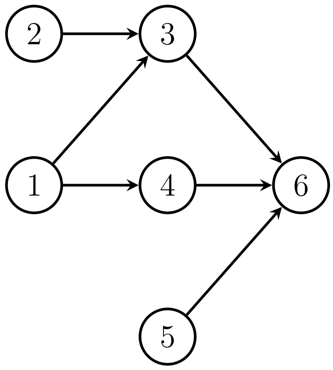
- You should download and extract cvx from `http://cvxr.com/cvx/`. Afterwards, you should execute cvx_setup in Matlab.
- A small cvx-example in Matlab is given below

```
cvx_begin
  variable p(m)
  maximize(sum(p))
  subject to
    p <= 1
cvx_end
```

- Matlab-variables may be used in the cvx-environment.
- You need to use the cvx-function **entr(x)** for calculating the entropy, because this function is known to be convex within cvx. **entr(X)** for a matrix is evaluated componentwise.

- Please scroll -

Exercise 2. (Optimizing processor speed) Consider the example for optimizing processor speed in section 4.2 of the lecture. Let $f(s) = s^2$.



(a) Give the minimal completion times $t_i^{\min}$ in dependence on the minimal processing times $\tau_i^{\min}$.

(b) Give the maximal interval for the energy restriction $E_{\max}$ in dependence on the demand vector $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6)$ in which the maximal completion time might change.

Let $s_{\min} = 1$ (GHz), $s_{\max} = 4$ (GHz), $\boldsymbol{\alpha} = (1, 1, 3, 2, 4, 5)$ the demand for processing each task and $E_{\max} = 24$.

(c) Determine the assignments of processor speeds and completion times by means of the cvx-package in Matlab.

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```
cvx_begin
  variable p(m)
  maximize(sum(p))
  subject to
    p <= 1
cvx_end
```

- Matlab-variables may be used in the cvx-environment.
- In cvx inv_pos(x) $= 1/x$, $x > 0$ is known to be convex.