Homework 12 in Optimization in Engineering

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Exercise 1. (Backtracking line search) Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ be a strict convex function with $\nabla^2 f(\boldsymbol{x}) \leq M \boldsymbol{I}_n$ for $M > 0$ and $\Delta\boldsymbol{x}$ the descent direction at $\boldsymbol{x} \in \mathbb{R}^n$.

(a) Show that the backtracking line search stopping criterion holds for

$$0 < t \leq -\frac{\nabla f(\boldsymbol{x})^T \Delta\boldsymbol{x}}{M \left\|\Delta\boldsymbol{x}\right\|_2^2} .$$

(b) Use the above result to derive an upper bound on the number of backtracking iterations.

Exercise 2. (Pure Newton method) Consider the minimization of the following functions. Plot f, g and their derivatives. Apply the pure Newton method for fixed step size $t = 1$ and calculate the values for the first few iterations. Calculate the difference to the minimum in each iteration.

(a) The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ with $f(x) = \log\left(e^x + e^{-x}\right)$ has a unique minimizer $x^* = 0$. Use the starting values $x^{(0)} = 1$ and in a second run $x^{(0)} = 1.1$.

(b) The function $g : \mathbb{R}_{>0} \longrightarrow \mathbb{R}$ with $g(x) = -\log(x) + x$ has a unique minimizer $x^* = 1$. Use the starting value $x^{(0)} = 3$.

Hint: Note that $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ for $x \in \mathbb{R}$.