Homework 14 in Optimization in Engineering

Prof. Dr. Anke Schmeink, Michael Reyer, Alper Tokel

02.02.2015

Exercise 1. (Barrier method) Let

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & 2 \leq x \leq 4 \end{array}$$

be an optimization problem with $f(x) = x + 1$, $x \in \mathbb{R}$. The feasible set is $[2, 4]$ and the optimal solution $x^* = 2$. Formulate the logarithmic barrier function $\Phi(x)$ and calculate the optimal solution $x^*(t)$ of the problem

$$\text{minimize} \quad t f(x) + \Phi(x)$$

with $x \in \mathbb{R}$ and constant $t > 0$. Illustrate the development of $x^*(t)$ and $f(x^*(t))$ for increasing t. What happens for $t \to \infty$?

- Please scroll -

Exercise 2. (General Barriers) The log barrier is based on the approximation of the indicator function $I_{-}(u)$ with the logaritmic function $-(1/t)\log(-u)$ (Section 7.2.1 in the lecture notes). We can also construct barriers from other approximations, which in turn yield generalizations of the central path and barrier method. Let $h : \mathbb{R} \longrightarrow \mathbb{R}$ be a twice differentiable, closed, increasing convex function with $\textbf{dom}\ h = \mathbb{R}_{<0}$. One such function is $h(u) = \log(-u)$; another example is $h(u) = -1/u$ (for $u < 0$). Now consider the optimization problem (without equality constraints, for simplicity)

$$\begin{array}{ll} \text{minimize} & f_0(x) \\ \text{subject to} & f_i(x) < 0, \quad i = 1, \ldots, s, \end{array}$$

where f_i are twice differentiable. We define the h-barrier for this problem as

$$\Phi_h(x) = \sum_{i=1}^{s} h(f_i(x)),$$

with domain $\{x \mid f_i(x) < 0, i = 1, \ldots, s\}$. When $h(u) = -\log(-u)$, this is the usual logarithmic barrier; when $h(u) = -1/u$, Φ_h is called the inverse barrier. We define the h-central path as

$$x^*(t) = \text{argmin}\ t f_0(x) + \Phi_h(x),$$

where t>0 is a parameter.

(a) Explain why $t f_0(x) + \Phi_h(x)$ is convex in x, for each $t > 0$.

(b) Show how to construct a dual feasible $\boldsymbol{\lambda}$ from $x^*(t)$. Find the associated duality gap.

(c) For what functions h does the duality gap found in part (b) depend only on t and s?