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Tutorial 3

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Problem 1. (Separation Theorem) Complete the proof of the Separation Theorem (Theorem 2.1 in the lecture notes), which is proved for a special case in the lecture. Show that a separating hyperplane exists for two disjoint convex sets $\mathcal{C}$ and $\mathcal{D}$.

Hints.

- You can use the result proved in Theorem 2.1 in the lecture, i.e., that a separating hyperplane exists when there exist points in the two sets whose distance is equal to the distance between the two sets.
- If $\mathcal{C}$ and $\mathcal{D}$ are disjoint convex sets, then the set $\{\boldsymbol{x} - \boldsymbol{y} \mid \boldsymbol{x} \in \mathcal{C} \ , \ \boldsymbol{y} \in \mathcal{D}\}$ is convex and does not contain the origin.

Problem 2. (Supporting hyperplanes) Represent each of the following closed, convex sets $\mathcal{C} \subseteq \mathbb{R}^2$ as an intersection of halfspaces.

a) $\mathcal{C} = \{\boldsymbol{x} \in \mathbb{R}^2 \,|\, x_2 \geq e^{x_1}\}$.

b) $\mathcal{C} = \{\boldsymbol{x} \in \mathbb{R}^2_{>0} \,|\, x_1 x_2 \geq 1\}$.

Problem 3. (Converse supporting hyperplane theorem) Suppose the set $\mathcal{C}$ is closed, has nonempty interior, and has a supporting hyperplane at every point in its boundary. Show that $\mathcal{C}$ is convex.