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Tutorial 5

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Problem 1. (Products and quotients of convex functions) Let $f, g : \mathbb{R} \to \mathbb{R}_{>0}$. Prove the following statements.

a) If f and g are convex and not decreasing (or not increasing), then the product $p(x) = f(x)g(x)$ is convex.

b) If f and g are concave, f is not decreasing and g is not increasing (or vice versa), then the product $p(x) = f(x)g(x)$ is concave.

c) If f is convex and not decreasing as well as g is concave and not increasing, then the quotient $q(x) = \frac{f(x)}{g(x)}$ is convex.

Problem 2. (Scalar Composition) Define the parameters $a, b \in \mathbb{R}$ such that the function below is concave:

$$f:\ \boldsymbol{x} = [x, y]^T \in \mathbb{R}^2 \to \mathbb{R},\ f(\boldsymbol{x}) = (b-1)\exp\Big(-b(\frac{a}{2}x^2 + \frac{a}{2}y^2 + 2xy)\Big).$$

Problem 3. (Maximum and minimum eigenvalues of a symmetric matrix)

Let $f(\mathbf{A}) = \lambda_{\max}(\mathbf{A})$ and $g(\mathbf{A}) = \lambda_{\min}(\mathbf{A})$ be functions which correspond, respectively, to the largest and smallest eigenvalues of the symmetric matrix $\mathbf{A} \in \mathcal{S}^m$. Prove the convexity or concavity of f and g.